Summary of 3D stress resolution

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Given $\mathbf{S}$ principal stress tensor with orientation $x'y'z'$

Rotate to N-S, E-W components

$$\mathbf{R} = \begin{bmatrix} l & l' & l'' \\ m & m' & m'' \\ n & n' & n'' \end{bmatrix} \text{ where } \begin{array}{rl} l & = x * x' \\ l' & = x * y' \\ l'' & = x * z' \\ \ldots & \end{array}$$

$$\mathbf{S}' = \mathbf{R}^T \mathbf{S} \mathbf{R}$$

And given plane with normal vector direction cosines $\mathbf{N}$

Traction $\mathbf{T} = \mathbf{S'} * \mathbf{N}$ (row and column multiplication)
$T = \sqrt{\mathbf{T}(1)^2 + \mathbf{T}(2)^2 + \mathbf{T}(3)^2}$ traction magnitude

$\sigma_n = \mathbf{T} \cdot \mathbf{N}$ dot product for normal traction magnitude

$\mathbf{B} = \mathbf{T} \times \mathbf{N}$ cross product for null vector
$B = \sqrt{\mathbf{B}(1)^2 + \mathbf{B}(2)^2 + \mathbf{B}(3)^2}$ $\mathbf{B}$ magnitude
$\mathbf{B}_{normalized} = \mathbf{B}./B$ normalize for orientation if necessary

$\mathbf{T_s} = \mathbf{N} \times \mathbf{B}$ cross product for shear traction vector
$\tau = \sqrt{\mathbf{T_s}(1)^2 + \mathbf{T_s}(2)^2 + \mathbf{T_s}(3)^2}$ shear traction magnitude
$\mathbf{T_s}_{normalized} = \mathbf{T_s}./\tau$ normalize for shear traction orientation

Coulomb failure function: $\Delta\sigma_f = \Delta\tau - (\mu - P)\Delta\sigma_n$