3D dislocations

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Fault surfaces as discontinuities (2D moving to 3D)

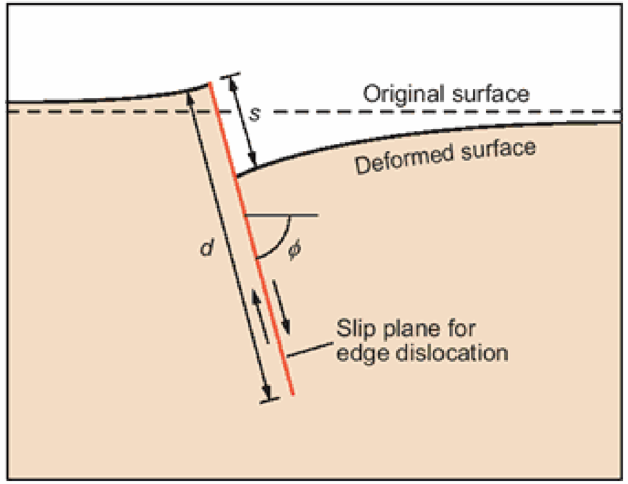


Figure 8.28 Mechanical model for normal fault using an edge dislocation in an elastic half-space. Refer to Section 5.6.4 for description of the edge dislocation. See Resor and Pollard (2012).

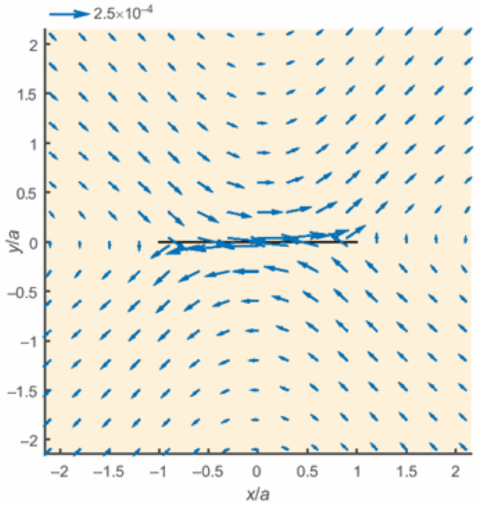


Figure 8.34 Normalized displacements, **u**/a, due to slip on the model fault using (8.15) with a positive driving stress, $\Delta\sigma_{II}$ = 1 MPa, and elastic constants G = 3,000 MPa and ν = 0.25. The greatest displacements in this field of view occur at the middle of the model fault where $|u_x/a| = 2.5 \times 10^{-4}$. Calculation from Pollard and Segall (1987).

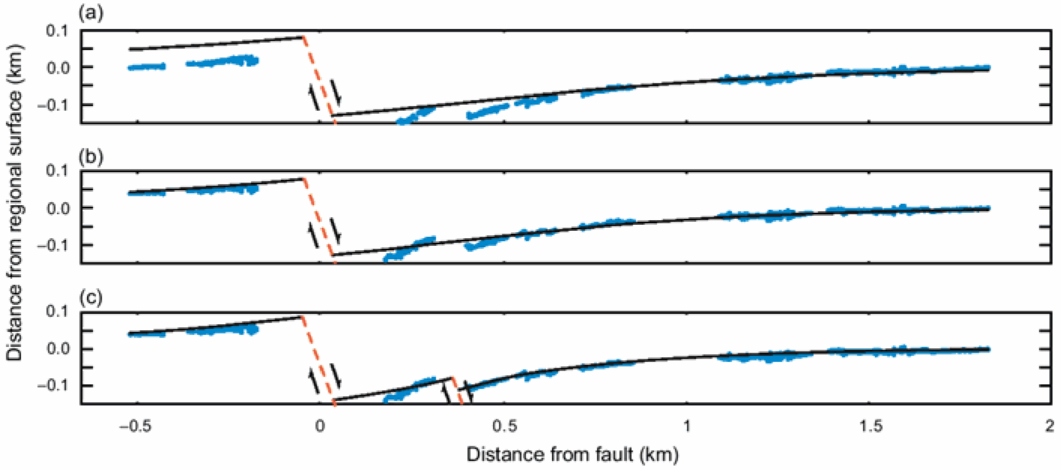
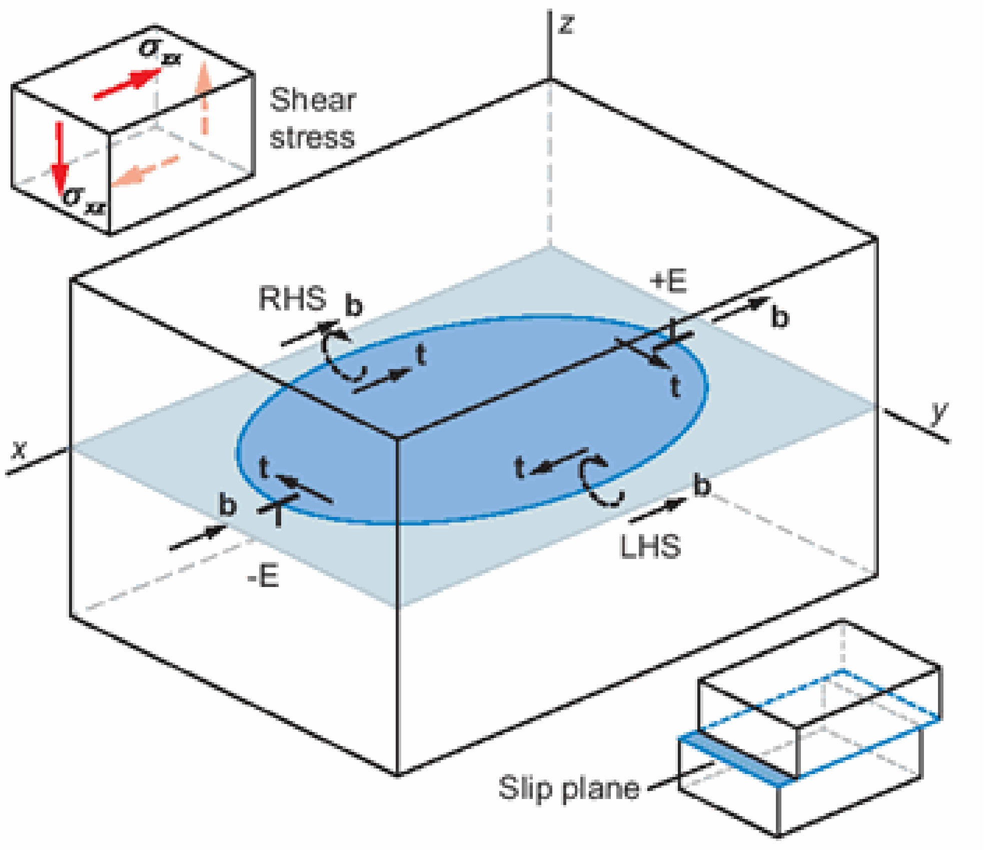
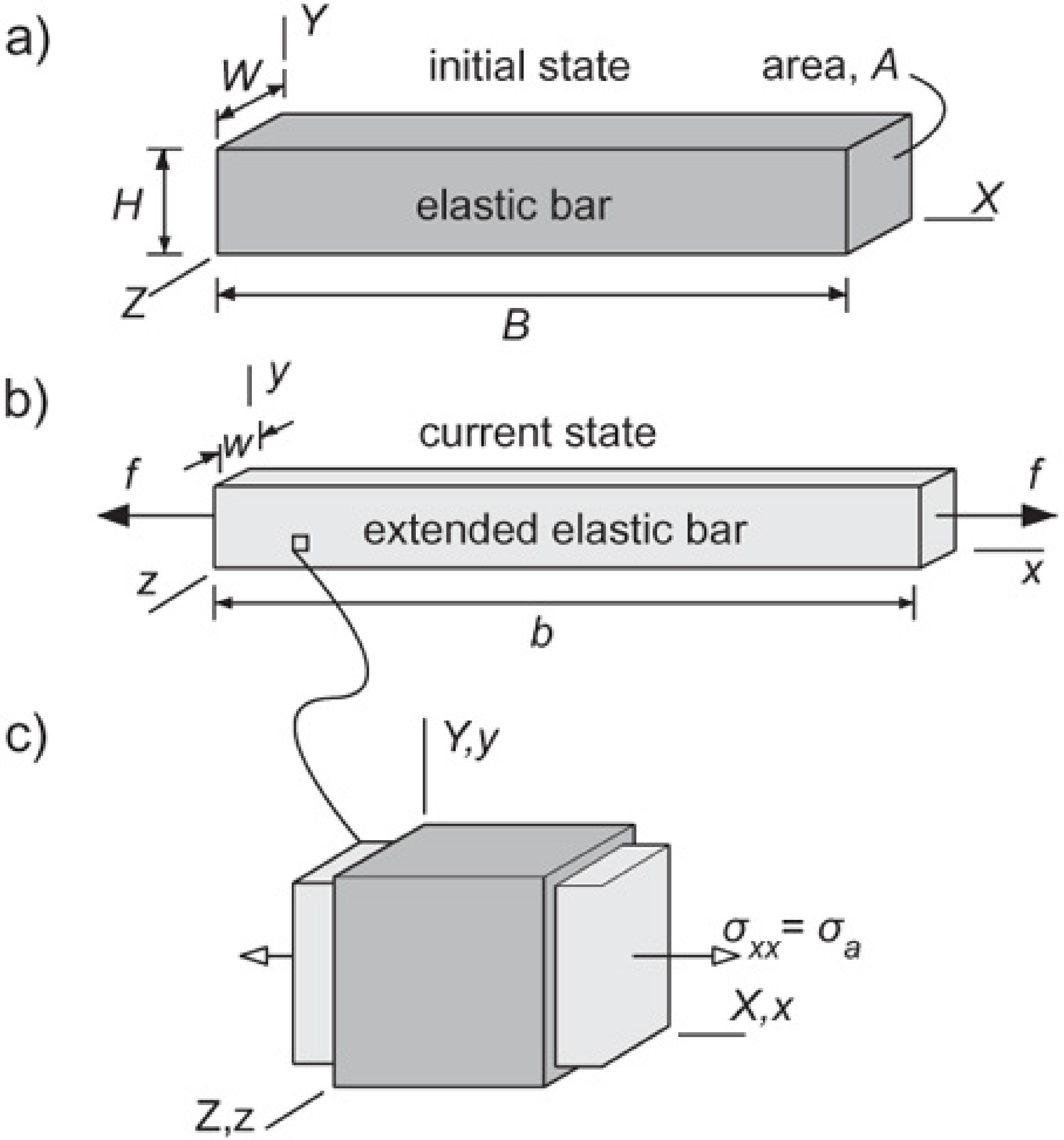


Figure 8.29 Elastic dislocation model displacements (black curve) compared to GPS data on structural elevations of upper Esplanade Formation (blue dots). (a) GPS data and single dislocation. (b) GPS data rotated 1° to account for regional tilt and single dislocation. (c) Rotated GPS data and two dislocations. Modified from Resor (2008), Figure 18.



Idealized Elastic Material

- Linear relationship between force and extension (Hooke, 1676):
  - C e l l l n o s s s t t u u
  - Ut tensio sic uis
  - As extension so the force



$\sigma_a = f/A$

$e_a = (b-B)/B$

$\sigma_p = 0$

$e_p = (w-W)/W$

$\nu = - e_p / e_a$

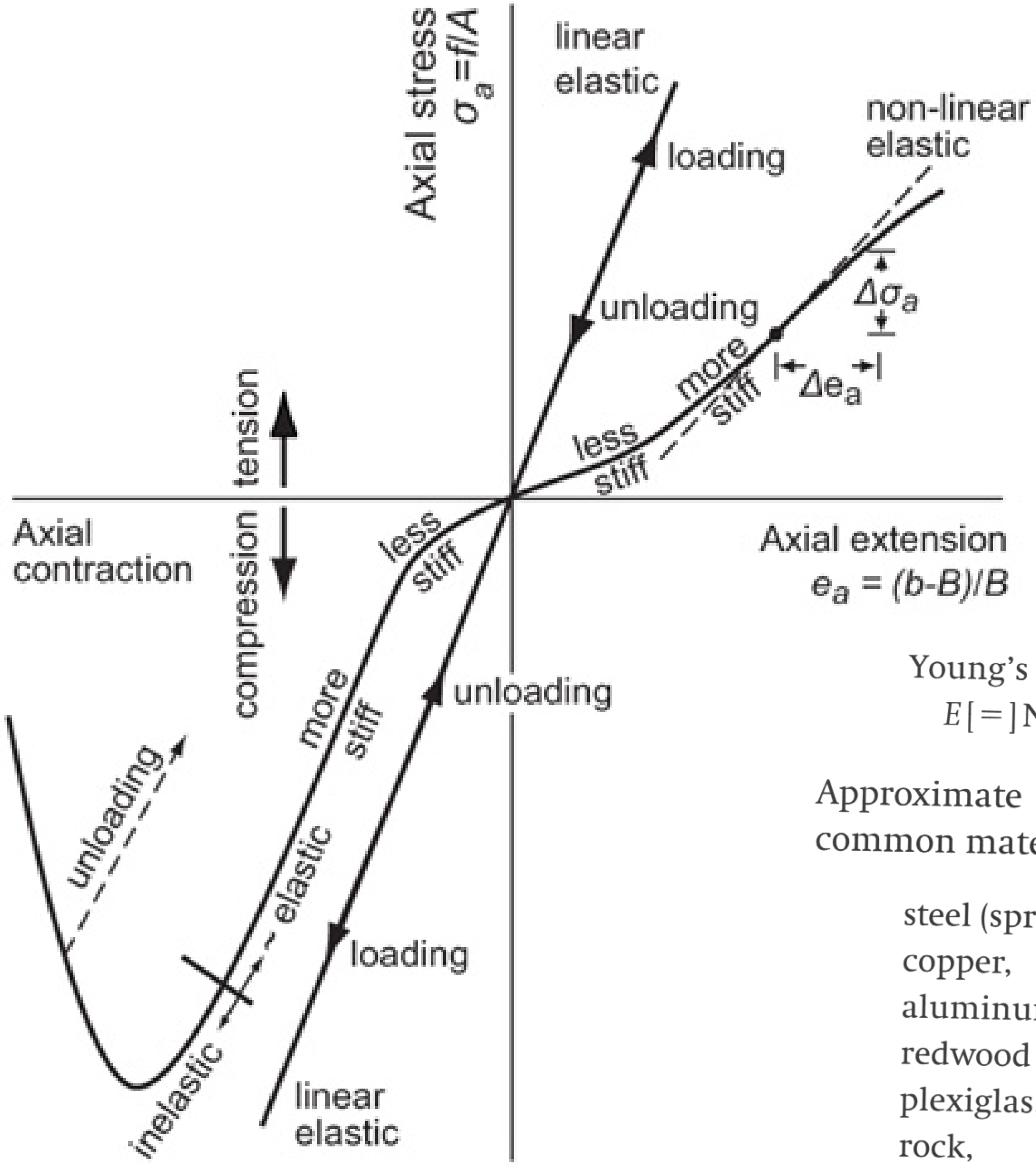
(Poisson ratio)

Measures compressibility

rubber, $\nu \sim 0.5$

cork, $\nu \sim 0.0$

rock, $\nu \sim 0.1$ to 0.3



E (young's modulus)
= d σ_a/d e_a
Units of stress

$$\text{Young's modulus, } E\,\{=\}\,\mathrm{ML^{-1}T^{-2}}, \quad \text{and} \quad E\,[=]\,\mathrm{Nm^{-2}} = \mathrm{Pa} \tag{8.8}$$

Approximate values of Young's modulus for common materials are (Eshbach, 1961):

steel (spring),	$E \sim 200$ GPa
copper,	$E \sim 110$ GPa
aluminum,	$E \sim 70$ GPa
redwood (dry),	$E \sim 9$ GPa
plexiglas,	$E \sim 3$ GPa
rock,	$E \sim 1$ to 100 GPa

Idealized Elastic Material

$$\sigma_1 = (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3, \tag{5.1}$$

$$\sigma_2 = \lambda\varepsilon_1 + (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_3, \tag{5.2}$$

$$\sigma_3 = \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2G)\varepsilon_3, \tag{5.3}$$

Any parameter that gives the ratio of one of the stress components to one of the strain components is generically called an "elastic modulus." The two elastic moduli appearing in (5.1)–(5.3), λ and G, are also known as the Lamé parameters. The parameter G is often denoted, particularly in mathematical elasticity treatments, by the symbol μ (λ and μ being the two Greek consonants in the surname of the French elastician who first developed the above equations, Gabriel Lamé). In order to avoid confusion with the coefficient of friction, however, we will use G. As will be shown below, G is the *shear modulus*, as it relates stresses to strains in a state of pure shear. If reference is made to the Lamé parameter (singular), this refers specifically to λ.

-Jaeger, Cooke, Zimmerman

Idealized Elastic Material

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right]$$

Elastic constants

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad G = \frac{E}{2(1+\nu)}, \quad K = \frac{E}{3(1-2\nu)};$$

Lame's constant
Describes effects of dilatation on tensile stress

Shear modulus
Relates shear strain to shear stress

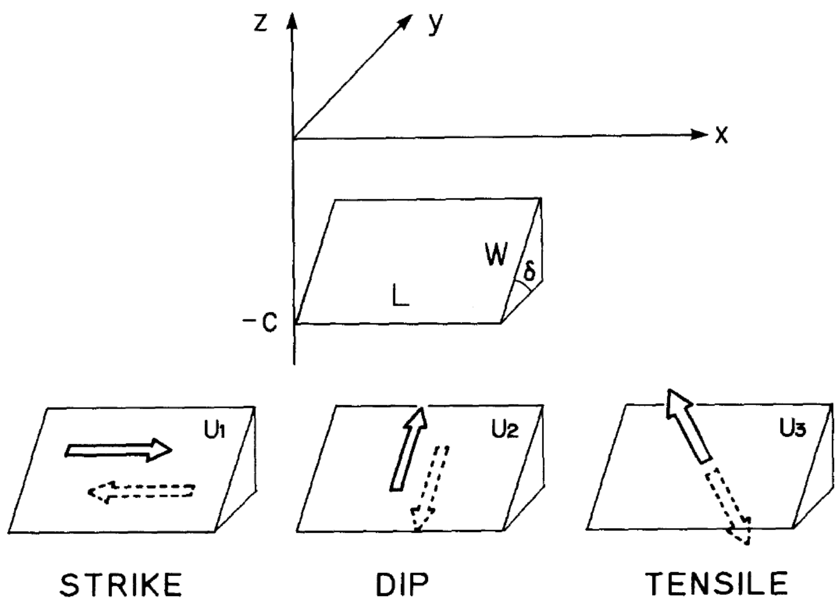
Bulk modulus
Relates volumetric strain to mean stress

E is Young's modulus which is the ratio of axial stress to axial strain
V is Poisson's ratio which is the negative of the ratio of transverse to longitudinal strain

--You only need two moduli to get the others

Okada, 1992 and 3D dislocations

- "Industry standard" for boundary element 3D elastic deformation modeling
- Linear elastic half space
- Rectangular elements
- Displacement boundary conditions
- Stress boundary conditions come from equivalent strain and displacement discontinuity



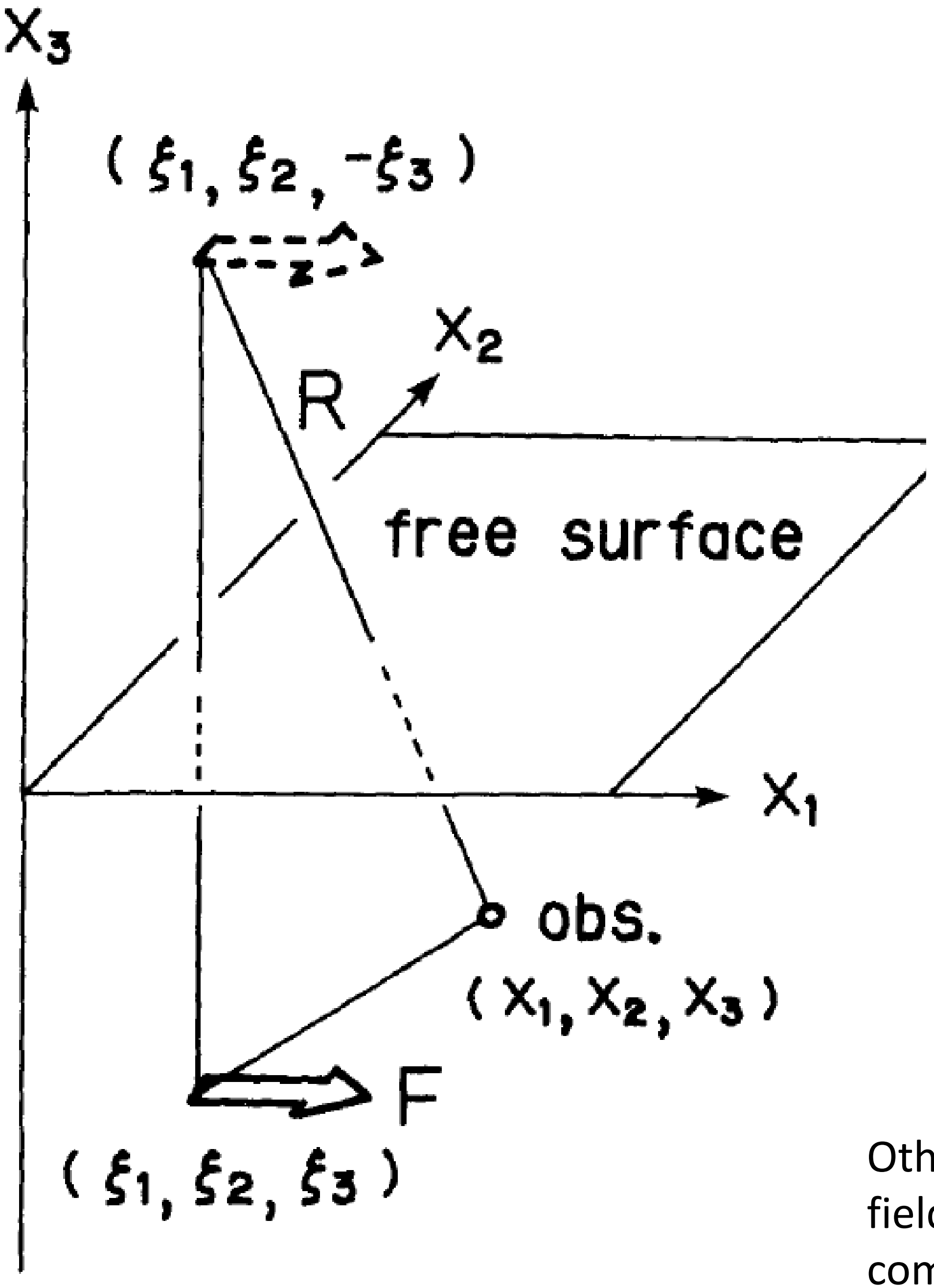


FIG. 1. A coordinate system adopted in this study.

$$u_i^j(x_1, x_2, x_3) = u_{iA}^j(x_1, x_2, -x_3) - u_{iA}^j(x_1, x_2, x_3) + u_{iB}^j(x_1, x_2, x_3) + x_3 u_{iC}^j(x_1, x_2, x_3) \qquad (1)$$

$$\begin{cases} u_{iA}^j = \dfrac{F}{8\pi\mu}\left\{(2-\alpha)\dfrac{\delta_{ij}}{R} + \alpha\dfrac{R_iR_j}{R^3}\right\} \\ u_{iB}^j = \dfrac{F}{4\pi\mu}\left\{\dfrac{\delta_{ij}}{R} + \dfrac{R_iR_j}{R^3} + \dfrac{1-\alpha}{\alpha}\left[\dfrac{\delta_{ij}}{R+R_3} + \dfrac{R_i\delta_{j3} - R_j\delta_{i3}(1-\delta_{j3})}{R(R+R_3)} - \dfrac{R_iR_j}{R(R+R_3)^2}(1-\delta_{i3})(1-\delta_{j3})\right]\right\} \\ u_{iC}^j = \dfrac{F}{4\pi\mu}(1-2\delta_{i3})\left\{(2-\alpha)\dfrac{R_i\delta_{j3} - R_j\delta_{i3}}{R^3} + \alpha\xi_3\left[\dfrac{\delta_{ij}}{R^3} - \dfrac{3R_iR_j}{R^5}\right]\right\}, \end{cases}$$

where, $\alpha = (\lambda + \mu)/(\lambda + 2\mu)$; λ and μ are Lamé's constants; δ_{ij} is the Kronecker delta; and $R_1 = x_1 - \xi_1$, $R_2 = x_2 - \xi_2$, $R_3 = -x_3 - \xi_3$, $R^2 = R_1^2 + R_2^2 + R_3^2$.

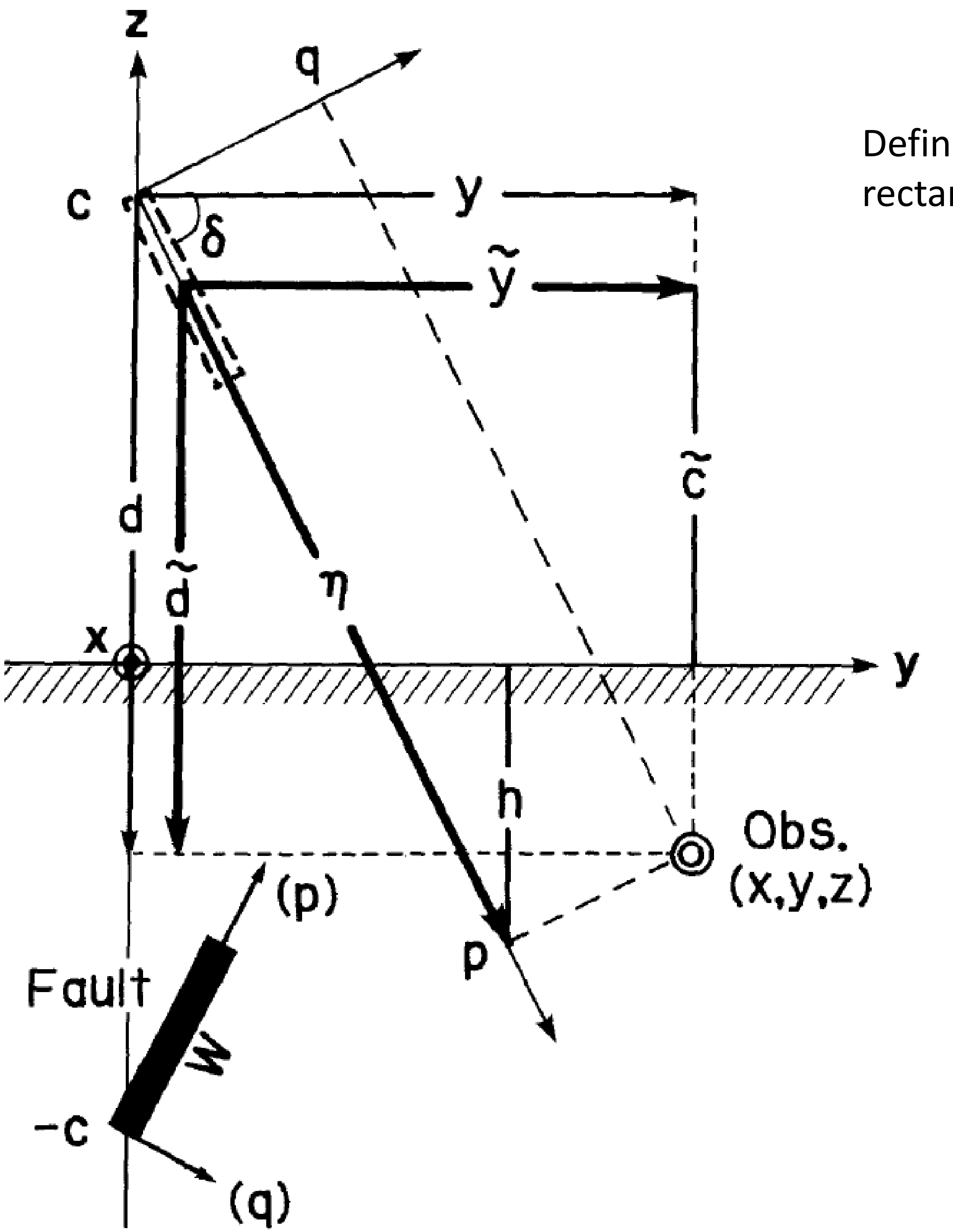
Displacement due to a point force
-ith component of displacement due to the point force in the jth direction

Part A: infinite medium terms
Part B: surface deformation related term
Part C: depth multiplied term

Other elastic field components

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right),$$

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij}.$$



Definition of geometry for rectangular source

Displacements due to discontinuity along finite rectangular source

--Integrate point sources along strike and dip

INTERNAL DISPLACEMENT FIELD DUE TO A FINITE RECTANGULAR SOURCE IN A HALF-SPACE. SEE TEXT AS TO THE MEANING OF THE TOP, MIDDLE, AND BOTTOM EQUATIONS IN EACH COMPARTMENT.

Displacement due to a Finite Fault at $(0,0,-c;\,\delta,L,W,U)$

$$\begin{cases} u_x(x,y,z) = U/2\pi\left[\, u_1^A - \hat{u}_1^A + u_1^B + z\,u_1^C \,\right] \\ u_y(x,y,z) = U/2\pi\left[\left(u_2^A - \hat{u}_2^A + u_2^B + z\,u_2^C\right)\cos\delta - \left(u_3^A - \hat{u}_3^A + u_3^B + z\,u_3^C\right)\sin\delta\right] \\ u_z(x,y,z) = U/2\pi\left[\left(u_2^A - \hat{u}_2^A + u_2^B - z\,u_2^C\right)\sin\delta + \left(u_3^A - \hat{u}_3^A + u_3^B - z\,u_3^C\right)\cos\delta\right] \end{cases}$$

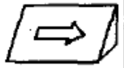
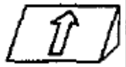
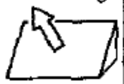
$d = c - z$ $R^2 = \xi^2 + \eta^2 + q^2$

$p = y\cos\delta + d\sin\delta$ $\tilde{y} = \eta\cos\delta + q\sin\delta$

$q = y\sin\delta - d\cos\delta$ $\tilde{d} = \eta\sin\delta - q\cos\delta$

$\alpha = (\lambda+\mu)/(\lambda+2\mu)$ $\tilde{c} = \tilde{d} + z$

$$u_i^A = f_i^A(\xi,\eta,z)\Big|_{\xi=x}^{\xi=x-L}\Big|_{\eta=p}^{\eta=p-W} \quad \hat{u}_i^A = f_i^A(\xi,\eta,-z)\Big\Vert \quad u_i^B = f_i^B(\xi,\eta,z)\Big\Vert \quad u_i^C = f_i^C(\xi,\eta,z)\Big\Vert$$

Type	f^A	f^B	f^C
Strike	$\frac{\Theta}{2} + \frac{\alpha}{2}\xi q Y_{11}$	$-\xi q Y_{11} - \Theta \quad -\frac{1-\alpha}{\alpha} I_1 \sin\delta$	$(1-\alpha)\,\xi Y_{11}\cos\delta \quad -\alpha\,\xi q Z_{32}$
U	$\frac{\alpha}{2}\frac{q}{R}$	$-\frac{q}{R} \quad +\frac{1-\alpha}{\alpha}\frac{\tilde{y}}{R+\tilde{d}}\sin\delta$	$(1-\alpha)\left[\frac{\cos\delta}{R} + 2qY_{11}\sin\delta\right] - \alpha\frac{\tilde{c}q}{R^3}$
	$\frac{1-\alpha}{2}\ln(R+\eta) - \frac{\alpha}{2}q^2 Y_{11}$	$q^2 Y_{11} \quad -\frac{1-\alpha}{\alpha} I_2 \sin\delta$	$(1-\alpha)\,qY_{11}\cos\delta \quad -\alpha\left[\frac{\tilde{c}\eta}{R^3} - zY_{11} + \xi^2 Z_{32}\right]$
Dip	$\frac{\alpha}{2}\frac{q}{R}$	$-\frac{q}{R} \quad +\frac{1-\alpha}{\alpha} I_3 \sin\delta\cos\delta$	$(1-\alpha)\frac{\cos\delta}{R} \quad - qY_{11}\sin\delta - \alpha\frac{\tilde{c}q}{R^3}$
U	$\frac{\Theta}{2} + \frac{\alpha}{2}\eta q X_{11}$	$-\eta q X_{11} - \Theta \quad -\frac{1-\alpha}{\alpha}\frac{\xi}{R+\tilde{d}}\sin\delta\cos\delta$	$(1-\alpha)\,\tilde{y}X_{11} \quad -\alpha\,\tilde{c}\eta q X_{32}$
	$\frac{1-\alpha}{2}\ln(R+\xi) - \frac{\alpha}{2}q^2 X_{11}$	$q^2 X_{11} \quad +\frac{1-\alpha}{\alpha} I_4 \sin\delta\cos\delta$	$-\tilde{d}X_{11} - \xi Y_{11}\sin\delta - \alpha\,\tilde{c}\left[X_{11} - q^2 X_{32}\right]$
Tensile	$-\frac{1-\alpha}{2}\ln(R+\eta) - \frac{\alpha}{2}q^2 Y_{11}$	$q^2 Y_{11} \quad -\frac{1-\alpha}{\alpha} I_3 \sin^2\delta$	$-(1-\alpha)\left[\frac{\sin\delta}{R} + qY_{11}\cos\delta\right] - \alpha\left[zY_{11} - q^2 Z_{32}\right]$
U	$-\frac{1-\alpha}{2}\ln(R+\xi) - \frac{\alpha}{2}q^2 X_{11}$	$q^2 X_{11} \quad +\frac{1-\alpha}{\alpha}\frac{\xi}{R+\tilde{d}}\sin^2\delta$	$(1-\alpha)\,2\xi Y_{11}\sin\delta + \tilde{d}X_{11} - \alpha\,\tilde{c}\left[X_{11} - q^2 X_{32}\right]$
	$\frac{\Theta}{2} - \frac{\alpha}{2}q(\eta X_{11} + \xi Y_{11})$	$q(\eta X_{11} + \xi Y_{11}) - \Theta - \frac{1-\alpha}{\alpha} I_4 \sin^2\delta$	$(1-\alpha)\left[\tilde{y}X_{11} + \xi Y_{11}\cos\delta\right] + \alpha\,q\left[\tilde{c}\eta X_{32} + \xi Z_{32}\right]$

$\Theta = tan^{-1}\frac{\xi\eta}{qR}$ $I_1 = -\frac{\xi}{R+\tilde{d}}\cos\delta - I_4\sin\delta$ $I_2 = \ln(R+\tilde{d}) + I_3\sin\delta$

$I_3 = \frac{1}{\cos\delta}\frac{\tilde{y}}{R+\tilde{d}} - \frac{1}{\cos^2\delta}\left[\ln(R+\eta) - \sin\delta\ln(R+\tilde{d})\right]$ $\left(I_3 = \frac{1}{2}\left[\frac{\eta}{R+\tilde{d}} + \frac{\tilde{y}q}{(R+\tilde{d})^2} - \ln(R+\eta)\right] \text{ if } \cos\delta = 0\right)$

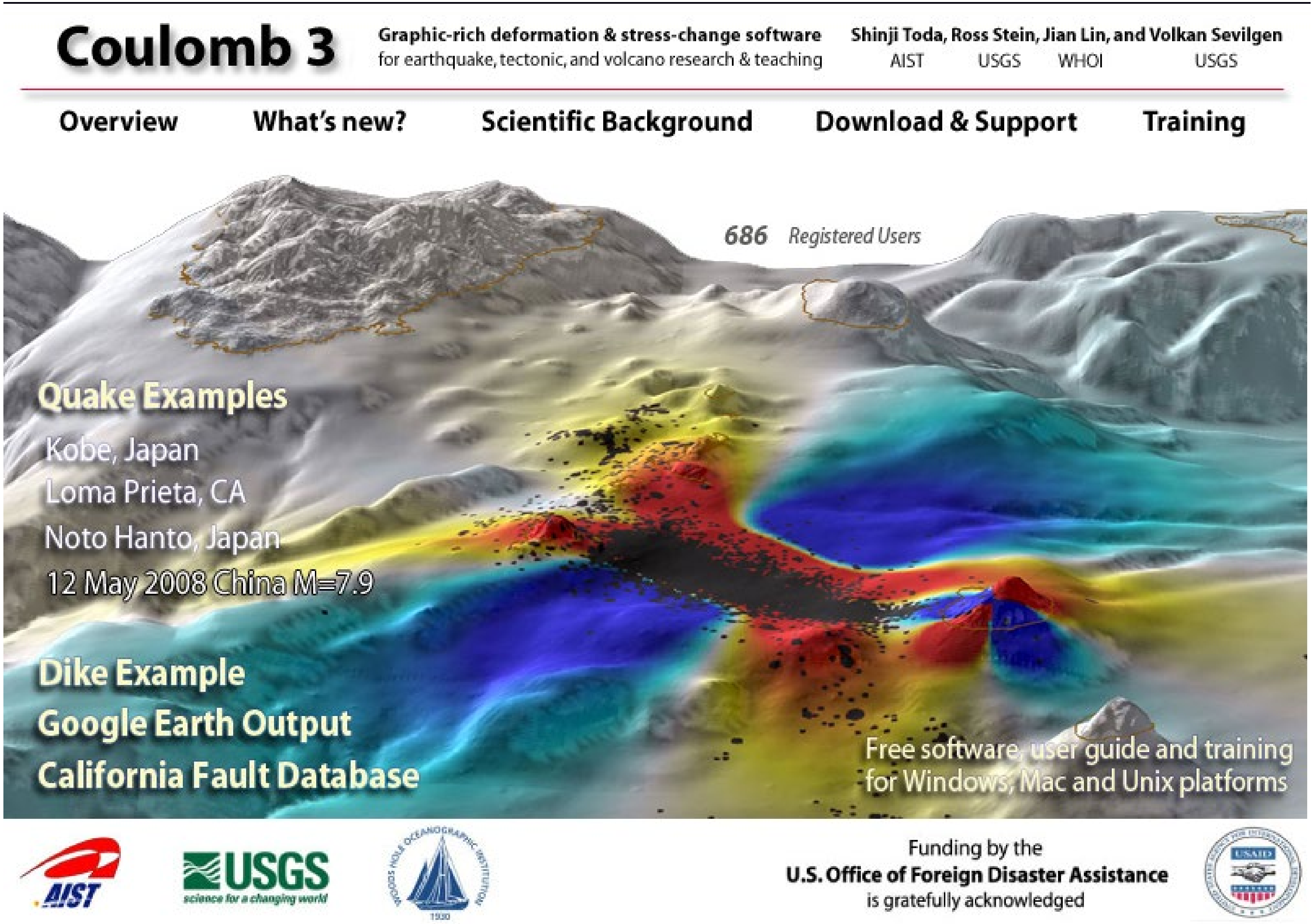
$X^2 = \xi^2 + q^2$ $I_4 = \frac{\sin\delta}{\cos\delta}\frac{\xi}{R+\tilde{d}} + \frac{2}{\cos^2\delta}\tan^{-1}\frac{\eta(X+q\cos\delta) + X(R+X)\sin\delta}{\xi(R+X)\cos\delta}$ $\left(I_4 = \frac{1}{2}\frac{\xi\tilde{y}}{(R+\tilde{d})^2} \text{ if } \cos\delta = 0\right)$

Strike direction

Dip direction (C: image)

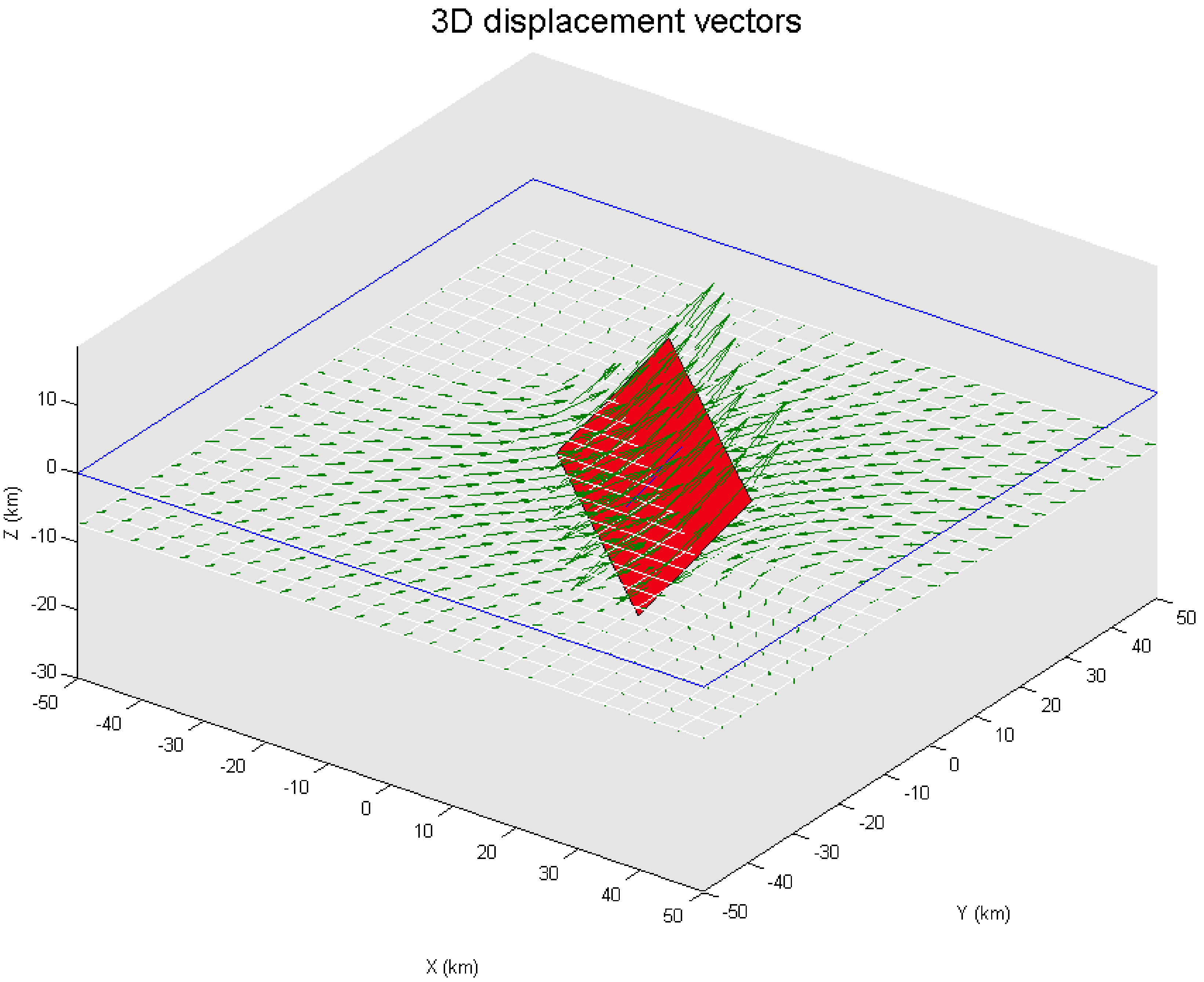
Opening direction (C: image)

Part A: infinite medium terms
Part B: surface deformation related term
Part C: depth multiplied term



http://quake.usgs.gov/research/deformation/modeling/coulomb/index.html

Coulomb



3D displacement vectors
10
0
-10
-20
-30
Z (km)
-50
-40
-30
-20
-10
0
10
20
30
40
50
X (km)
-50
-40
-30
-20
-10
0
10
20
30
40
50
Y (km)

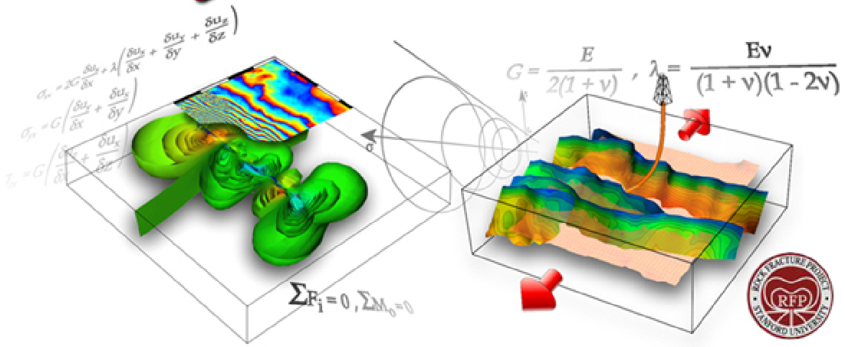
Earth Sciences

Structural Geology and Geomechanics

Software

- Program Description
- Who we are
- Rock Fracture Project
- Research

Poly3D and Poly3DGUI



Great Research Tool for Fracture Mechanics and Earthquake Studies

and

Industry's Choice for Geomechanical Analysis

of Fractured Oil and Gas Reservoirs

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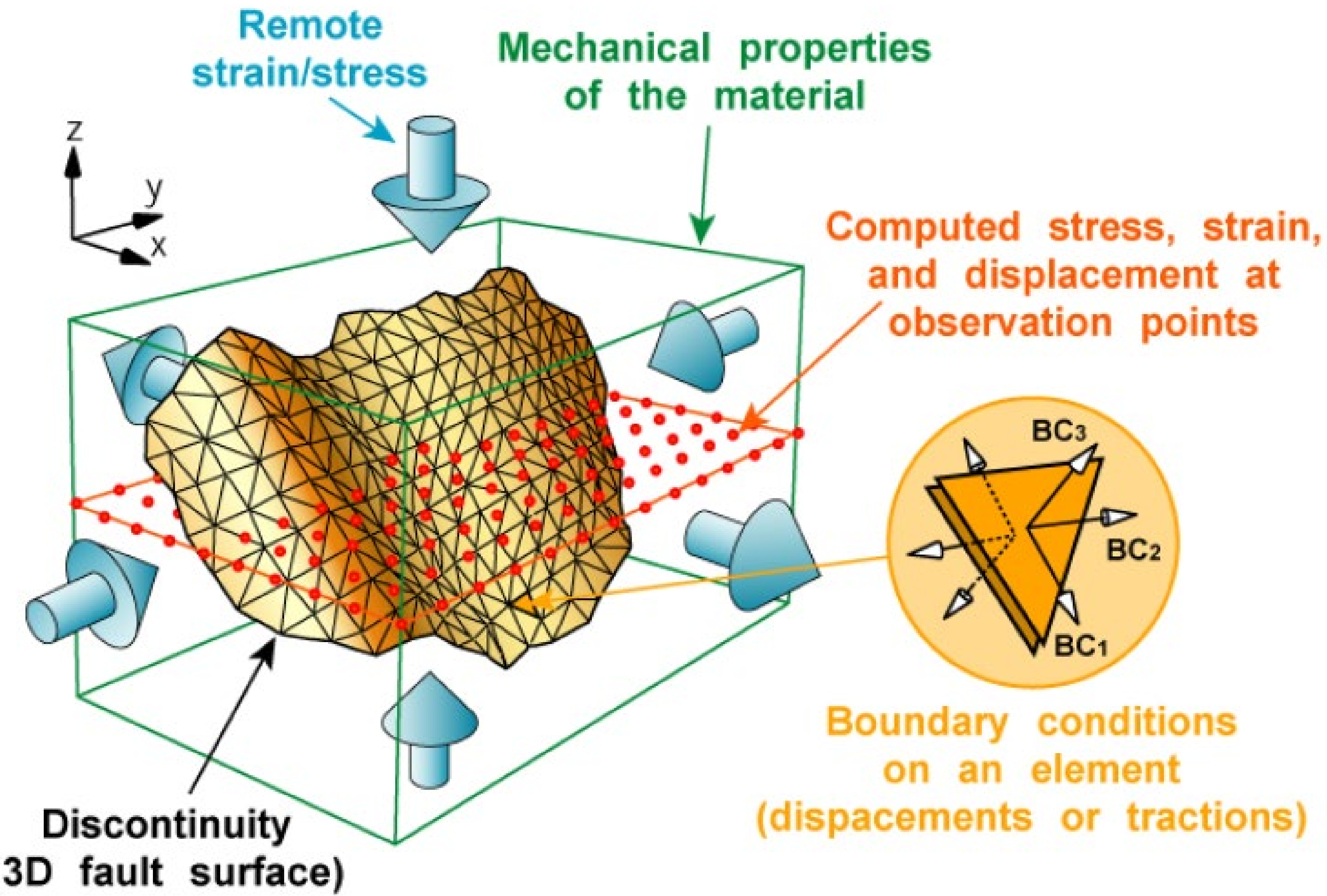
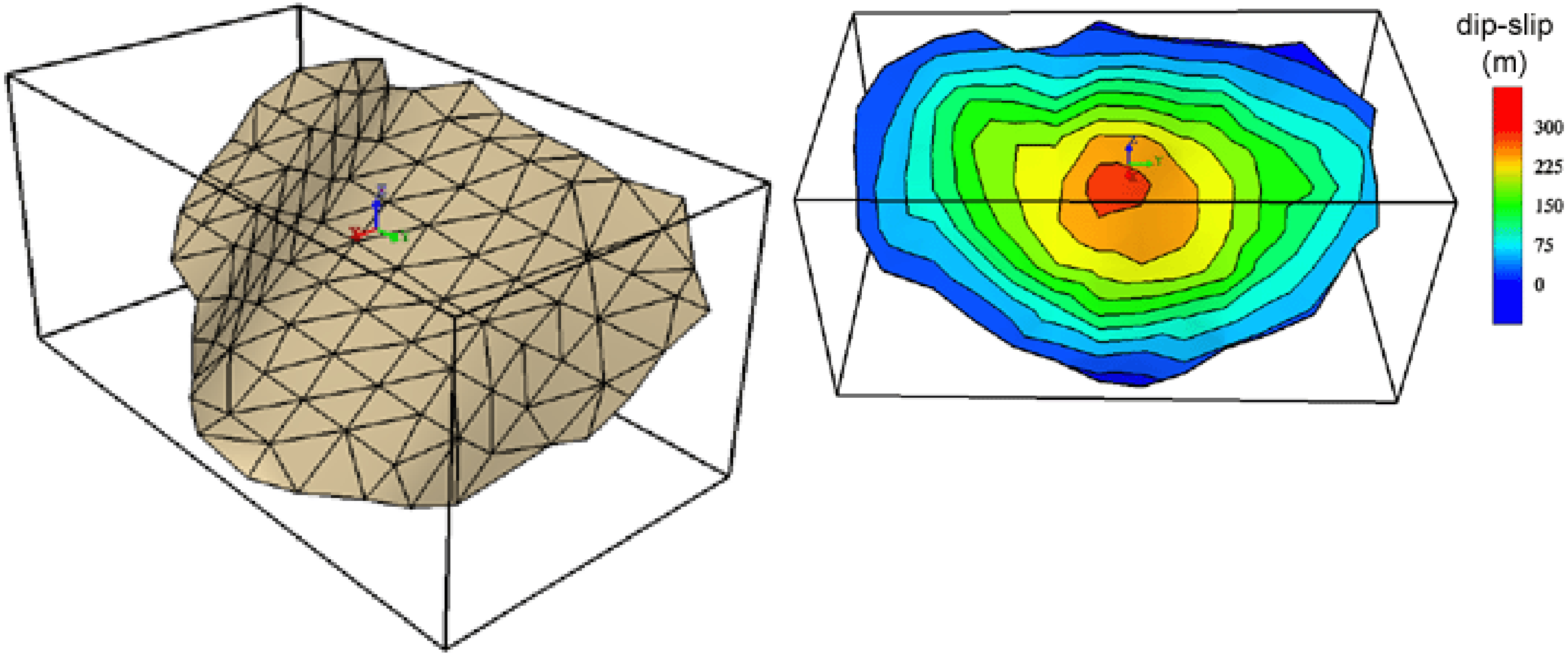
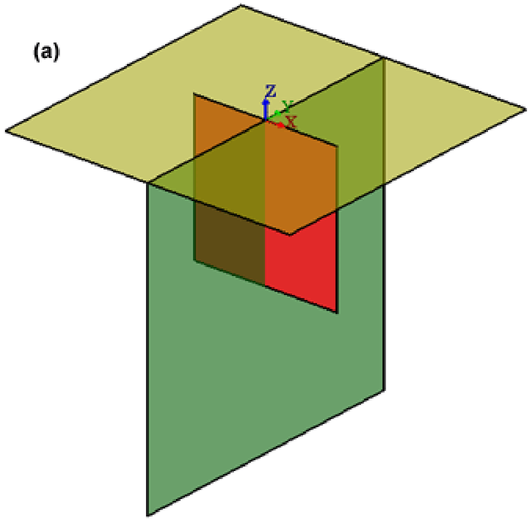
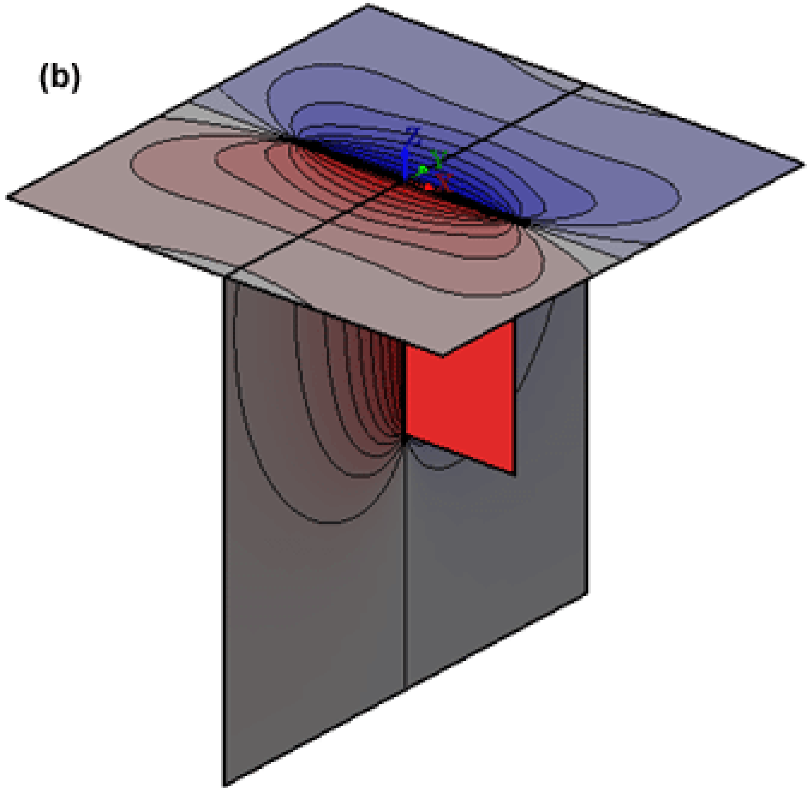


Figure 1: Poly3D model configuration for a 3D fault.

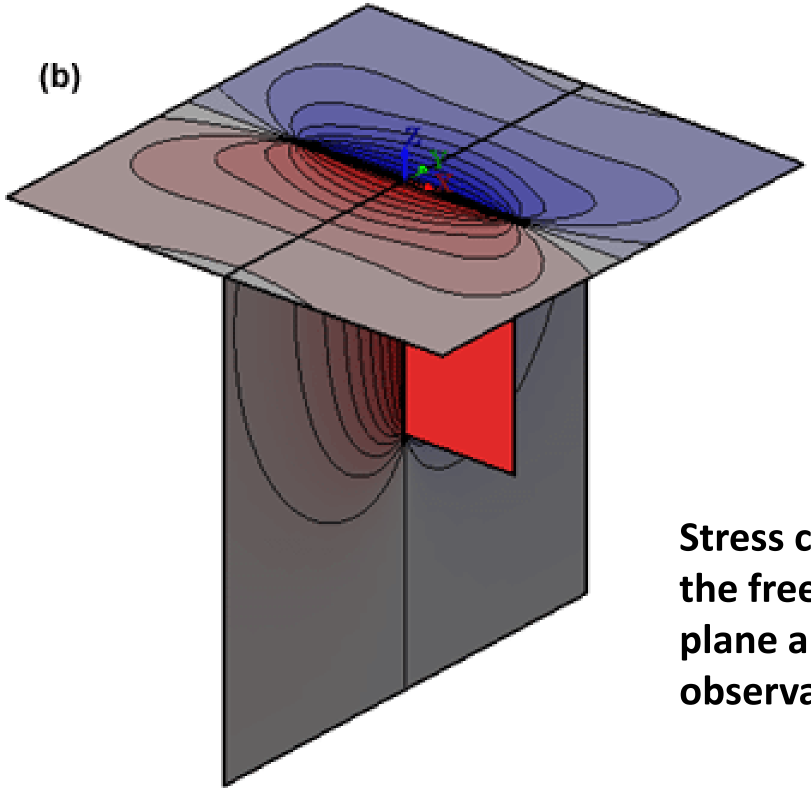


3D normal fault with constant stress drop

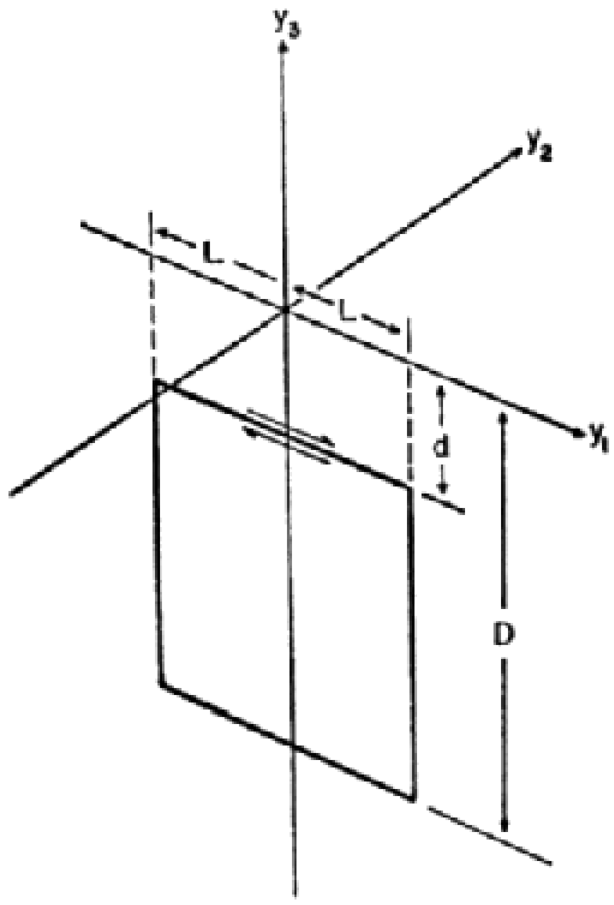




Poly3D color figure of the displacement Ux at both the free surface observation plane and the vertical observation plane



Stress component Sxy at both the free surface observation plane and the vertical observation plane



Chinnery’s fault