Advanced Structural Geology, Fall 2022

Faults and stress

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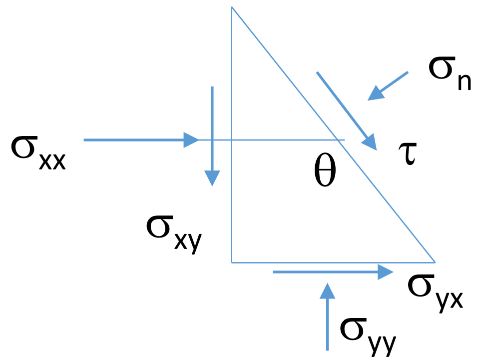
Relationship between traction and stress



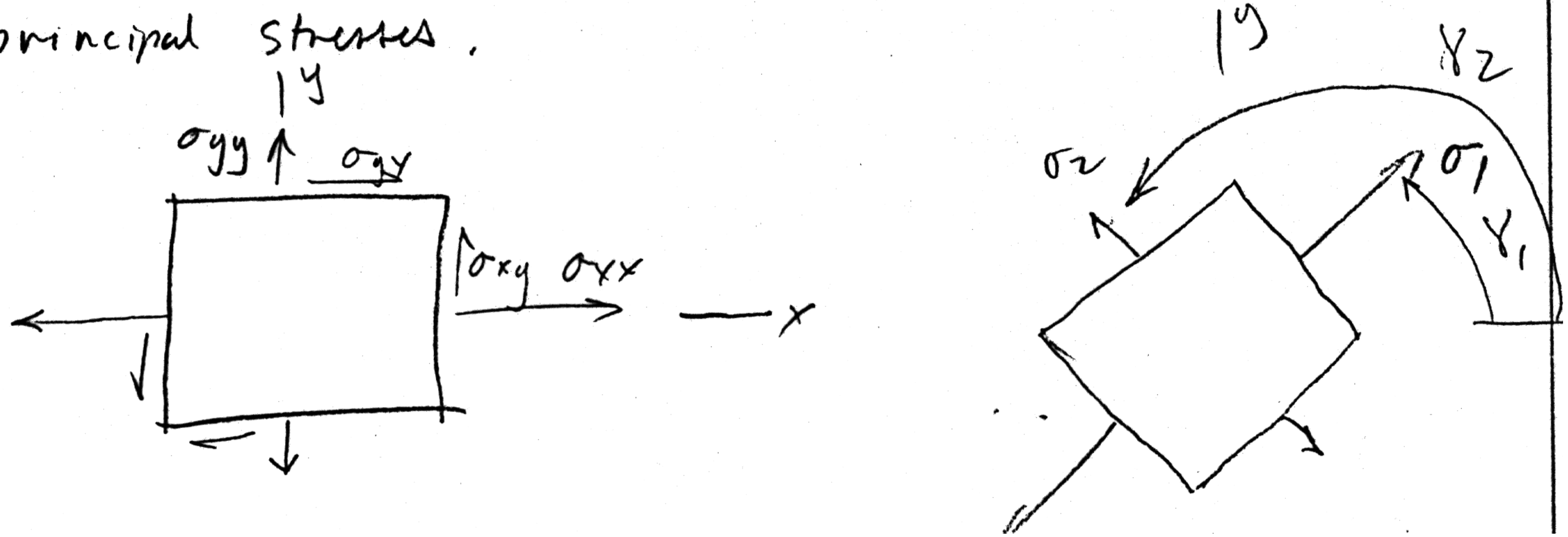
By balancing the traction with stress, we can solve for the normal and shear tractions as a function of the stress tensor components and the orientation of the plane.

Cauchy's equations:

$$\sigma_n = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\sigma_{xy}\cos\theta\sin\theta$$

$$\tau = \sigma_s = \sigma_{xy}\left(\cos^2\theta - \sin^2\theta\right) + \left(\sigma_{xx} - \sigma_{yy}\right)\cos\theta\sin\theta$$

Earth's surface is free of shear tractions, so normal stress on earth's surface from atmosphere is a principal stress. Other principal stresses are ⊥ so horizontal surfaces must contain the other two principal stresses.



Principal Stresses

Orientation:

$$\gamma_1 = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}}\right), \quad \gamma_2 = \gamma_1 + 90^\circ$$

Magnitude:

$$\sigma_1 = \frac{1}{2}(\sigma_{xx}+\sigma_{yy}) + \left[\frac{1}{4}(\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{xy})^2\right]^{1/2}$$

$$\sigma_2 = \frac{1}{2}(\sigma_{xx}+\sigma_{yy}) - \left[\frac{1}{4}(\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{xy})^2\right]^{1/2}$$

Can specify state of stress with the principal stresses + orientation

Principal Stresses

σ_1 is maximum compression
σ_3 is minimum (in 3D)
σ_2 is intermediate

Returning to Cauchy's equations:

$$\sigma_n = \sigma_{xx} \cos^2 \theta + \sigma_{yy} \sin^2 \theta + 2\sigma_{xy} \cos\theta \sin\theta$$

$$\tau = \sigma_s = \sigma_{xy}\left(\cos^2 \theta - \sin^2 \theta\right) + \left(\sigma_{xx} - \sigma_{yy}\right) \cos\theta \sin\theta$$

delete 0 components and use these identities:

$$\cos^2 \theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$\sin^2 \theta = \frac{1}{2} - \frac{1}{2}\sin 2\theta$$

and $\sigma_{xx} = \sigma_1$; $\sigma_{yy} = \sigma_3$

$$\cos\theta \sin\theta = \frac{1}{2}\sin 2\theta$$

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$$

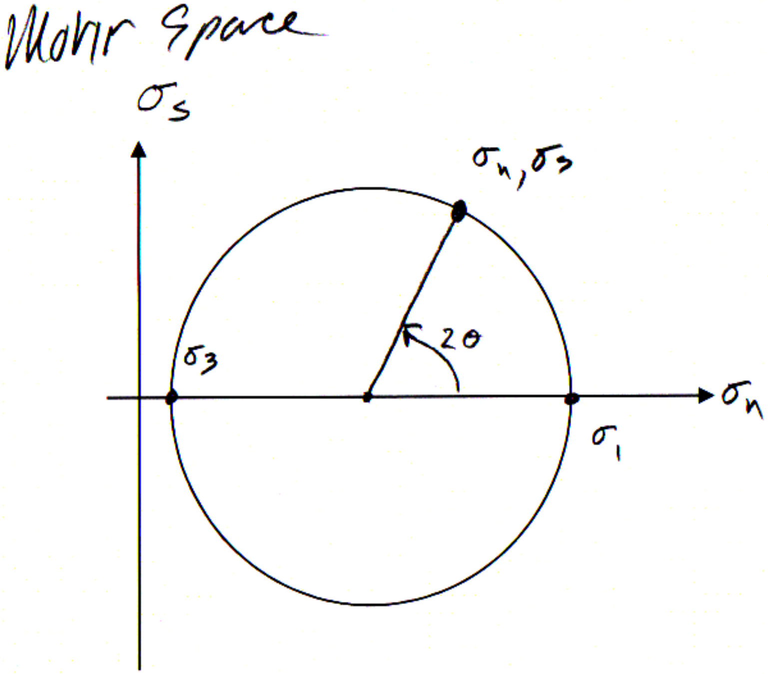
θ is positive clockwise from the σ_1 direction

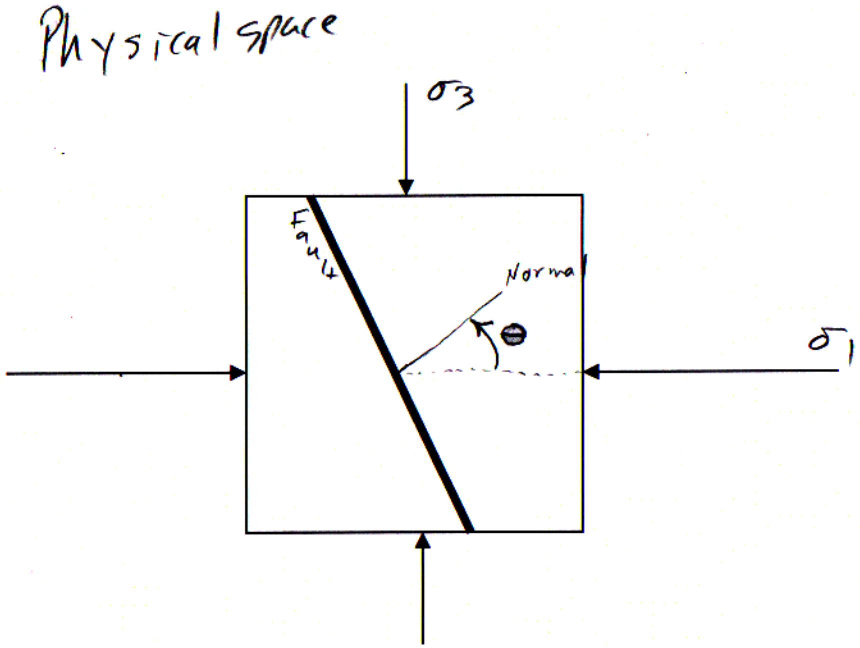
$$\tau = \sigma_s = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta$$

Faults, stress, and tractions

Mohr Circle

Graphical construction that lets us visualize the relationship between the principal stresses and tractions on a boundary (like a fault).





θ measured positive counter clockwise from σ_1 direction to normal of plane of interest

2θ measure positive clockwise from σ_n direction on Mohr circle

Christian Otto Mohr (October 8, 1835 – October 2, 1918) was a German civil engineer. In 1882, he famously developed the graphical method for analysing stress known as Mohr's circle and used it to propose an early theory of strength based on shear stress.

http://en.wikipedia.org/wiki/Christian_Otto_Mohr

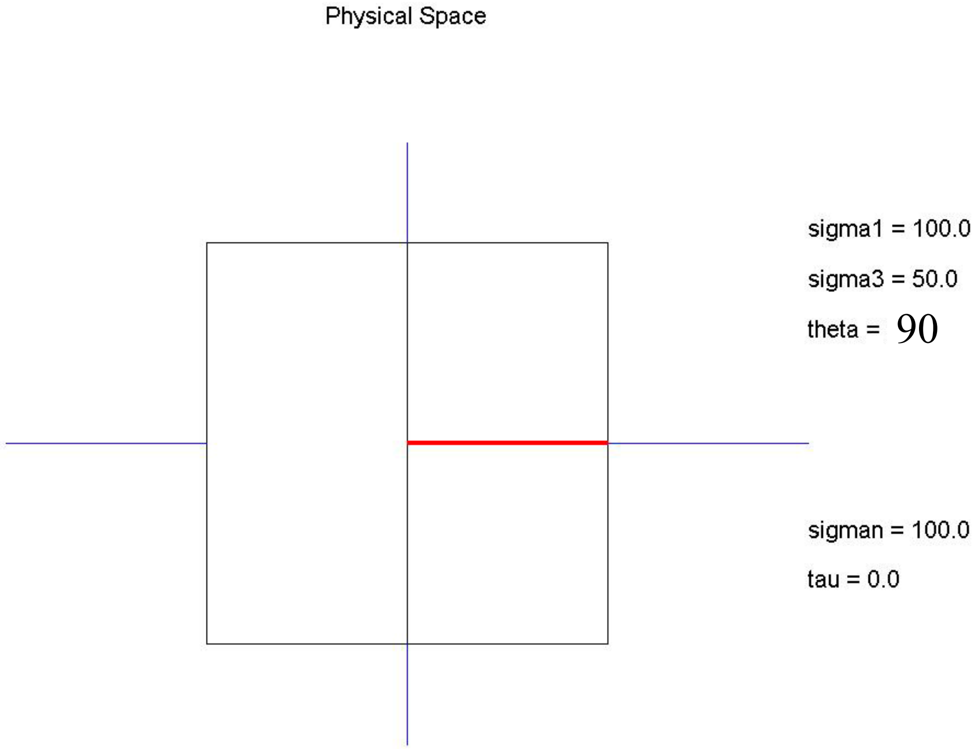
Faults and stress: traction and stress

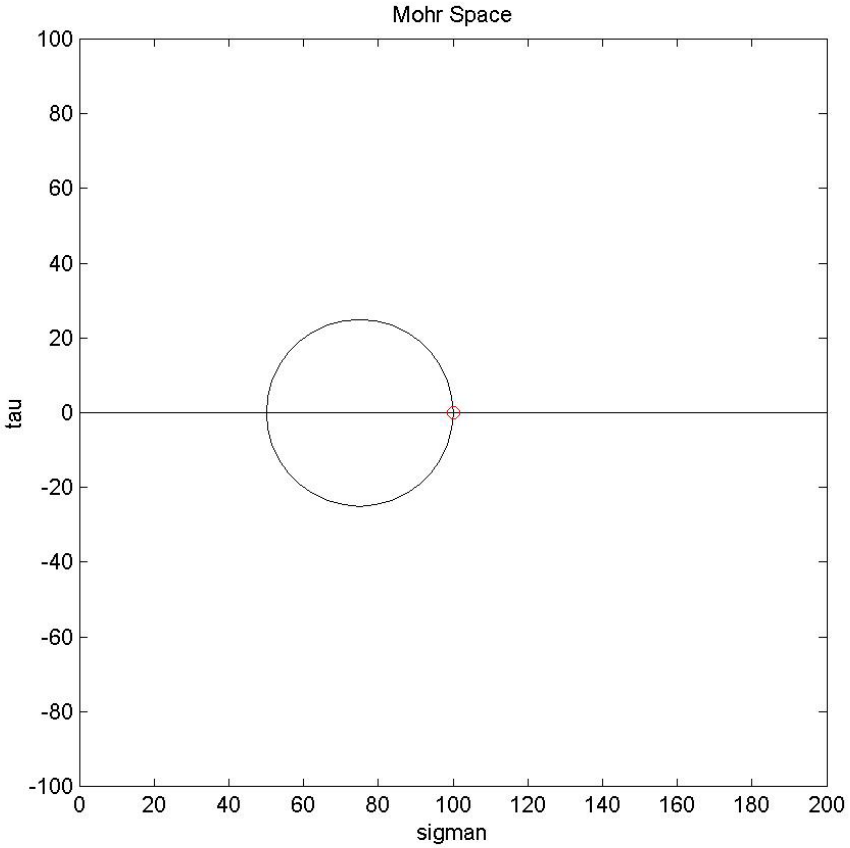
$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2}\cos 2\theta$$

$$\tau = \sigma_s = \frac{\sigma_1 - \sigma_3}{2}\sin 2\theta$$

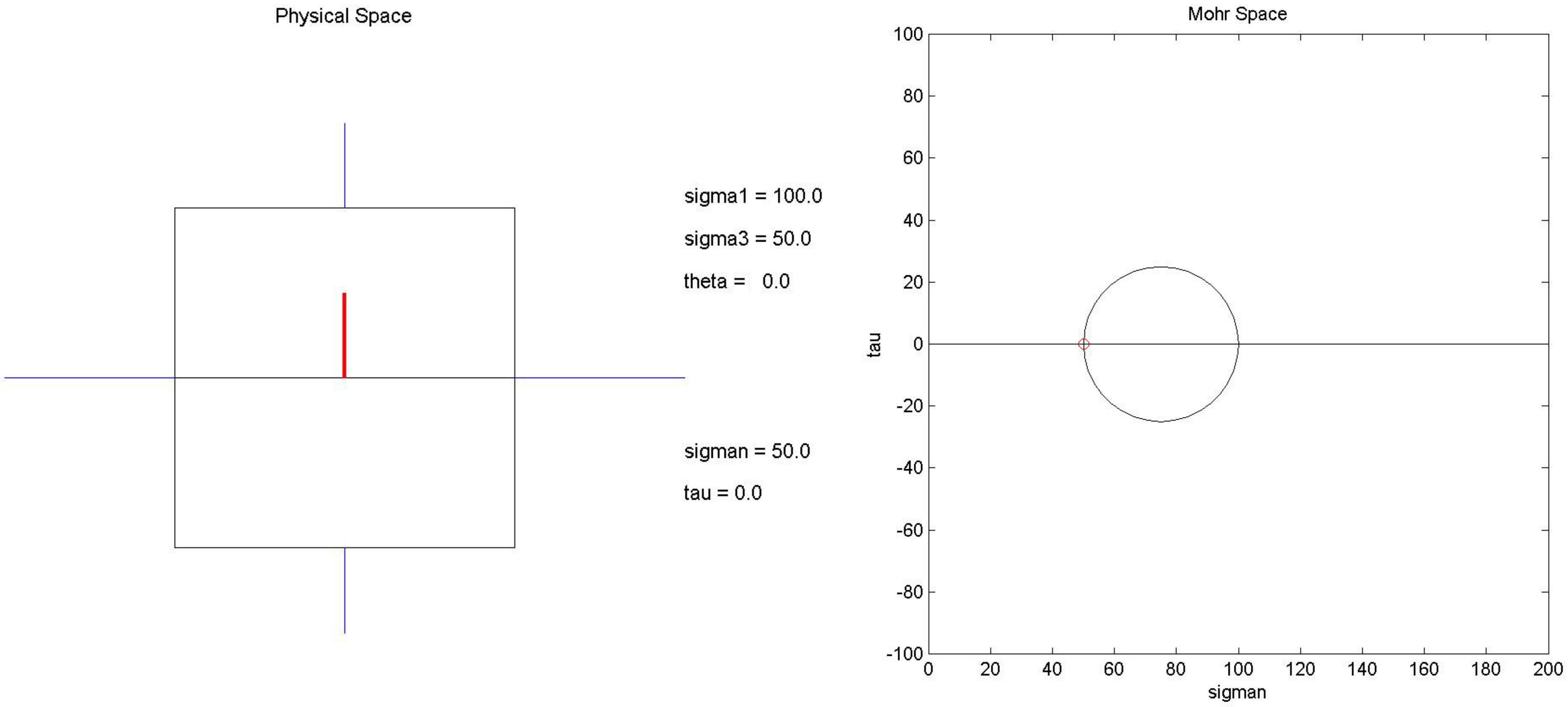
Equations of the Mohr Circle (also "Cauchy's equations")

Faults and stress: traction and stress

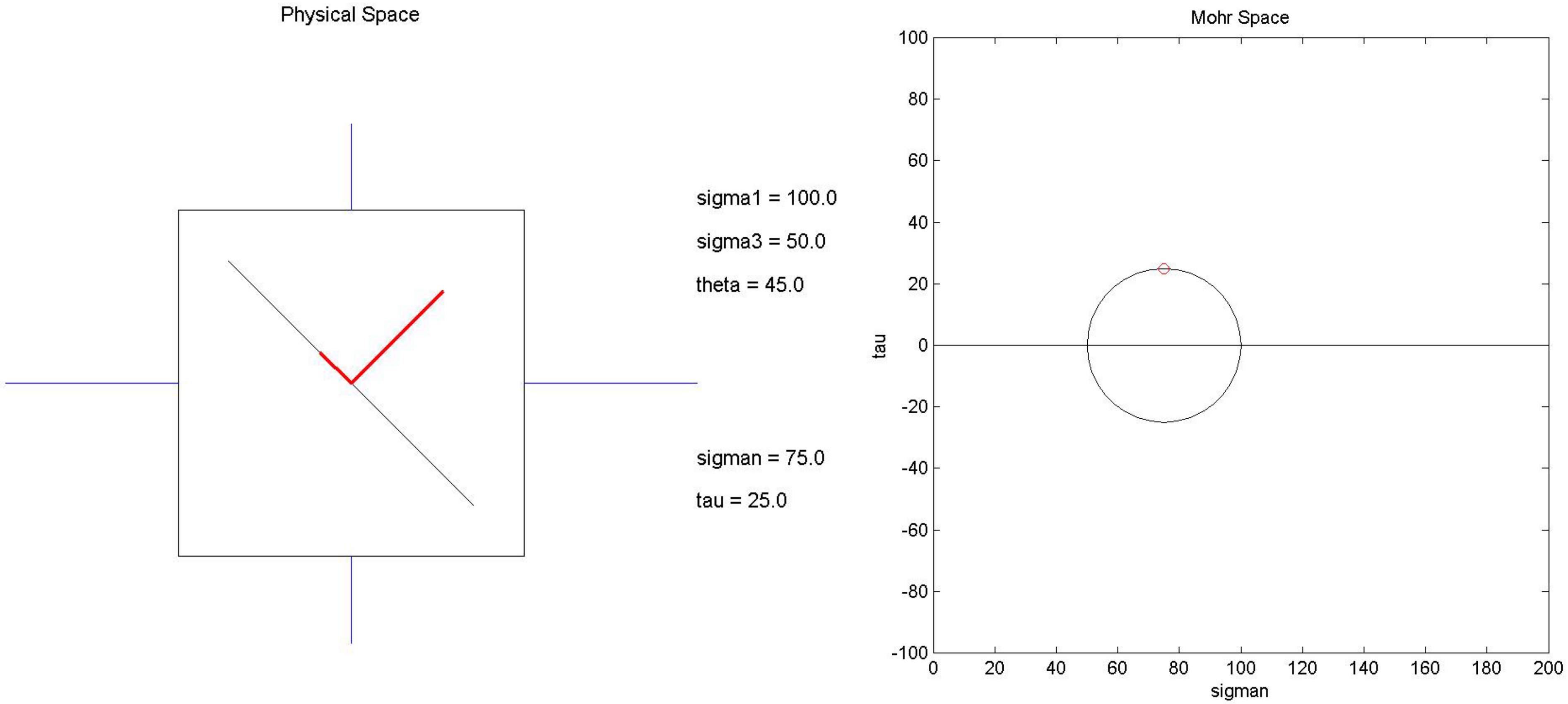




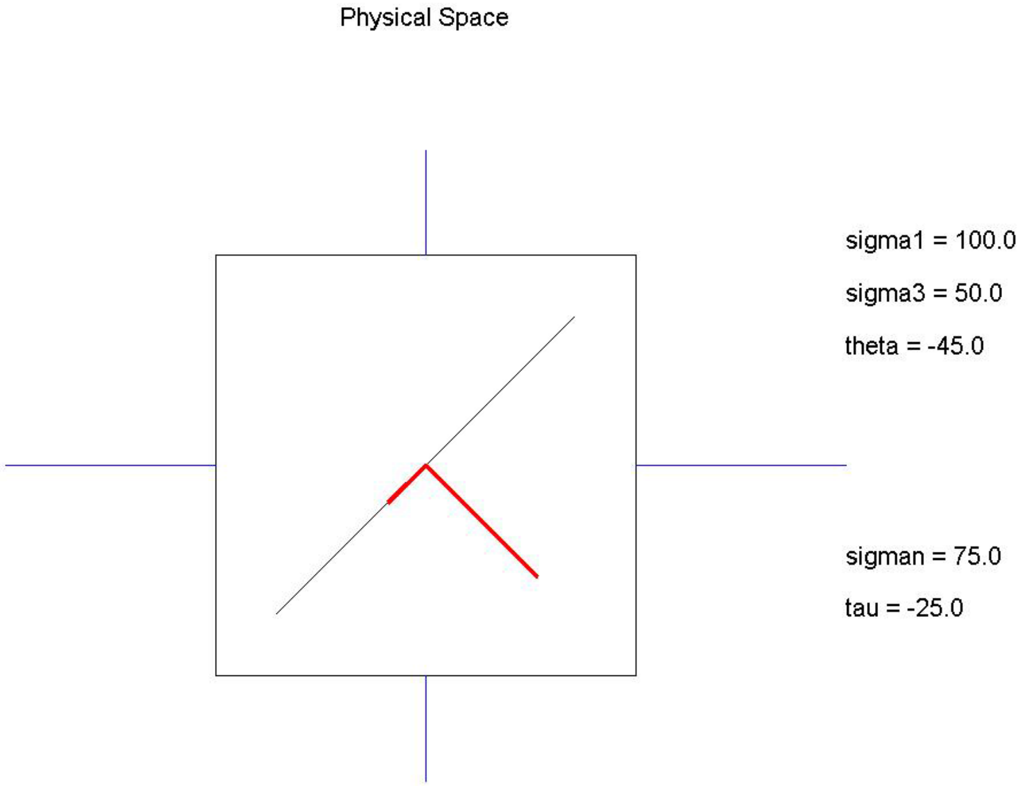
Faults and stress: traction and stress

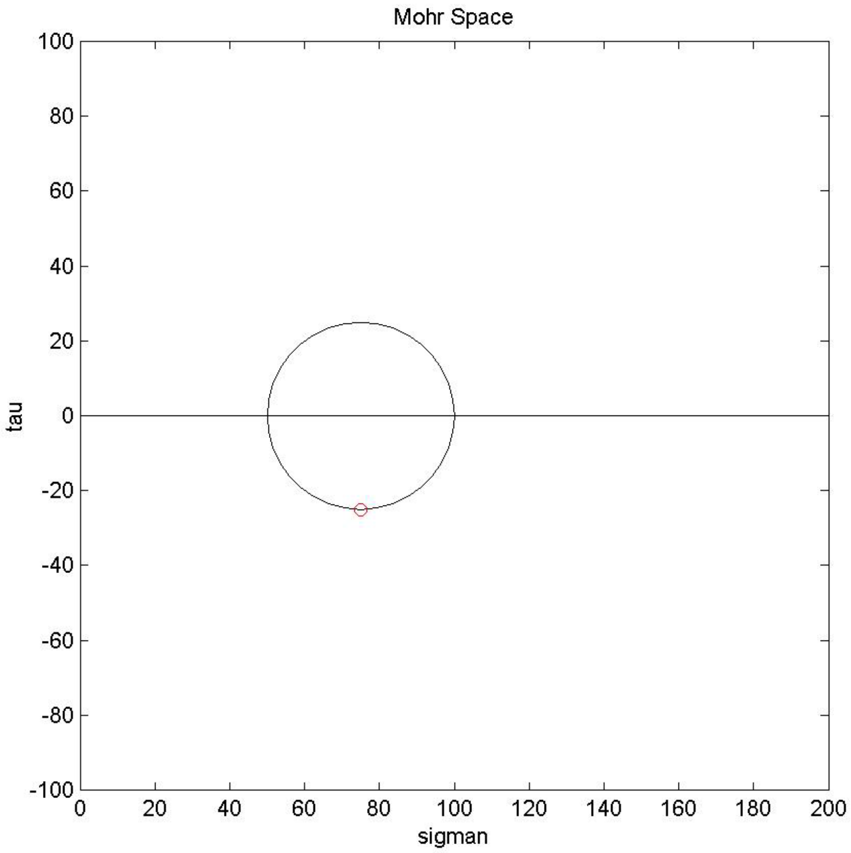


Faults and stress: traction and stress

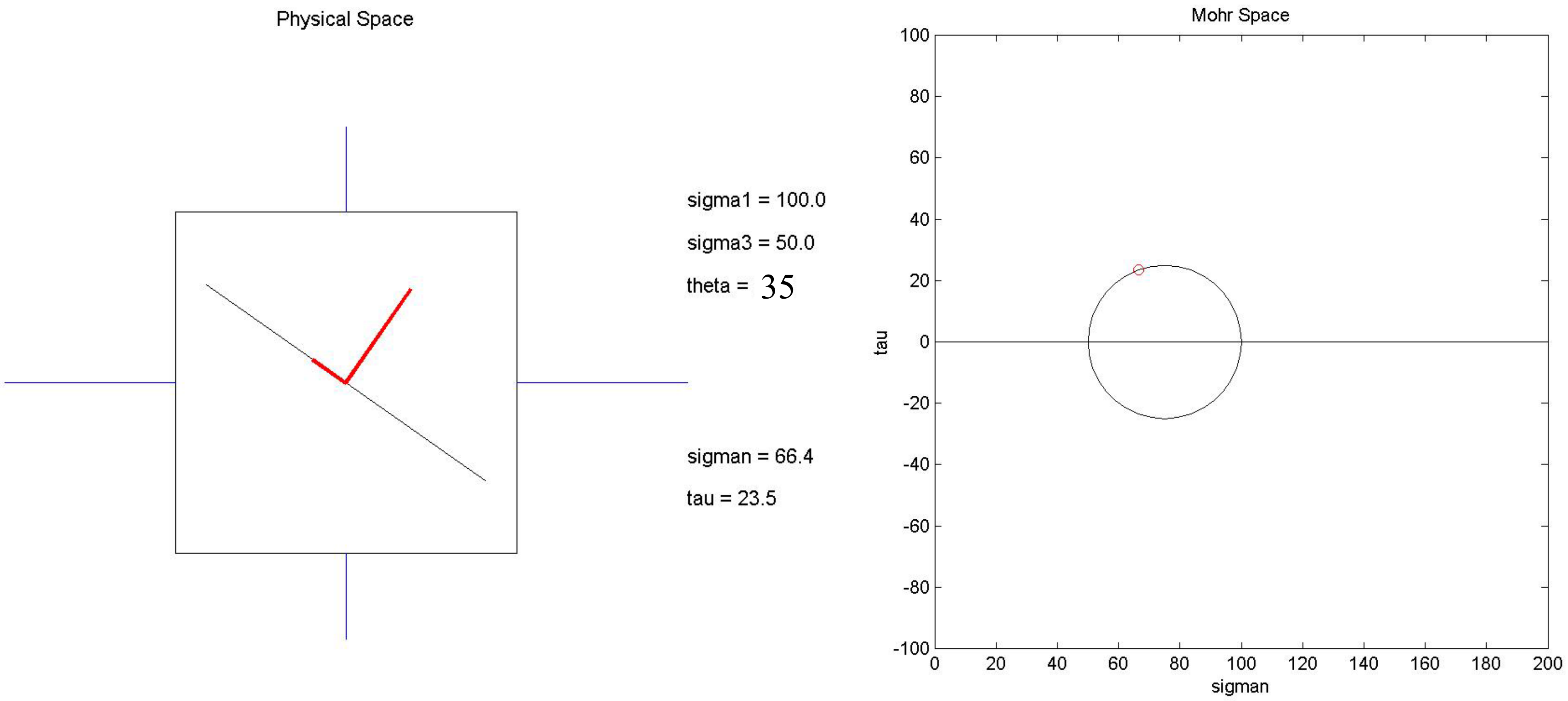


Faults and stress: traction and stress

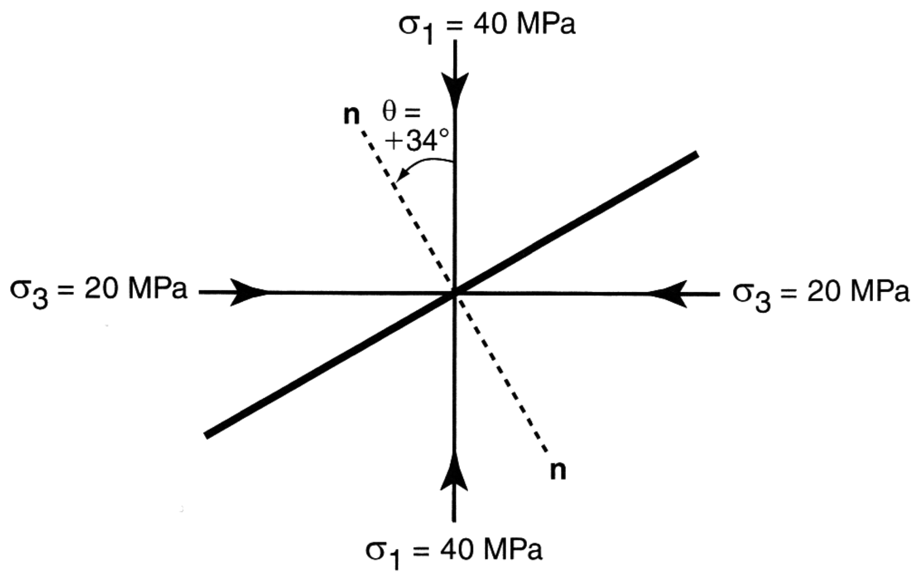




Faults and stress: traction and stress

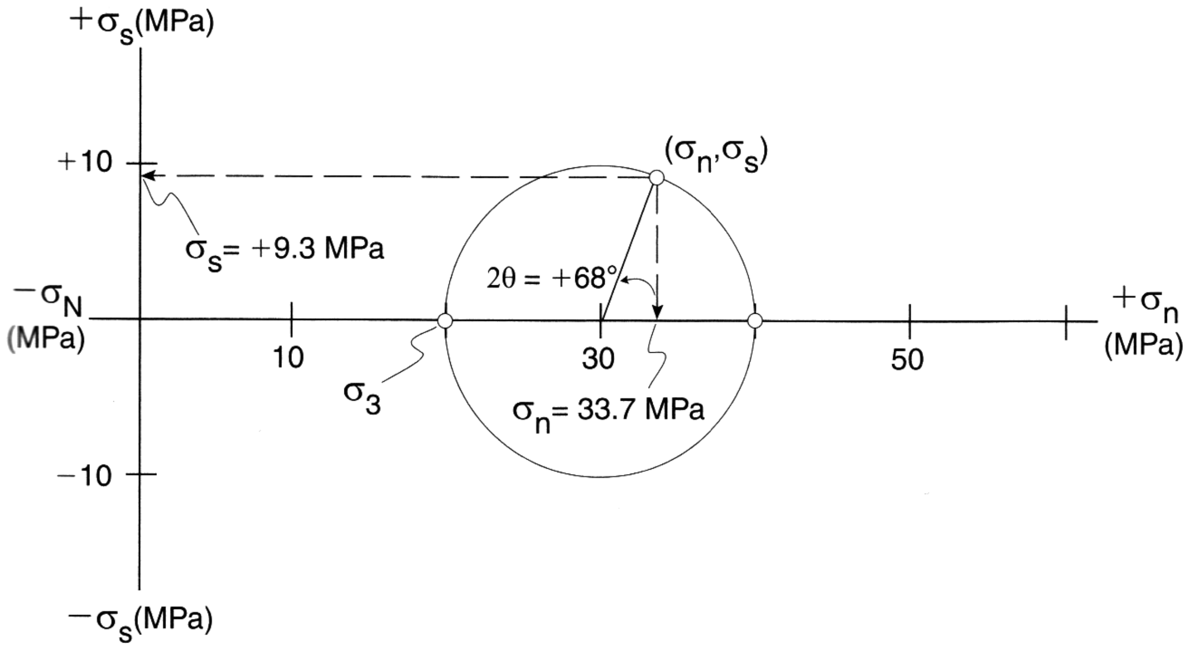


A

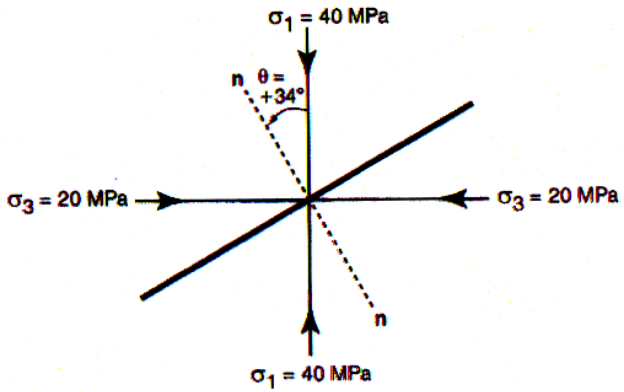


σ1 = 40 MPa
n
θ =
+34°
σ3 = 20 MPa
σ3 = 20 MPa
n
σ1 = 40 MPa

B



+σs(MPa)
+10
(σn,σs)
σs= +9.3 MPa
2θ = +68°
−σN
(MPa)
10
30
50
+σn
(MPa)
σ3
σn= 33.7 MPa
−10
−σs(MPa)



$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

$$\frac{40 + 20}{2} + \frac{40 - 20}{2} \cos 2 \cdot 34$$

$$\frac{60}{2} + \frac{20}{2} \cos 68$$

$$30 + 10 \cdot 0.375$$

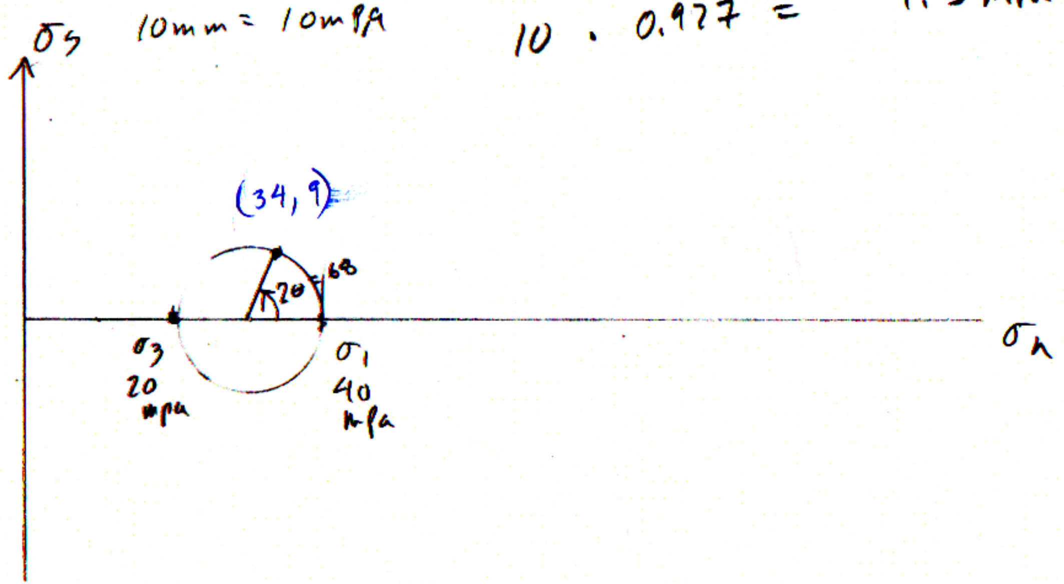
$$30 + 3.75 = 33.75$$

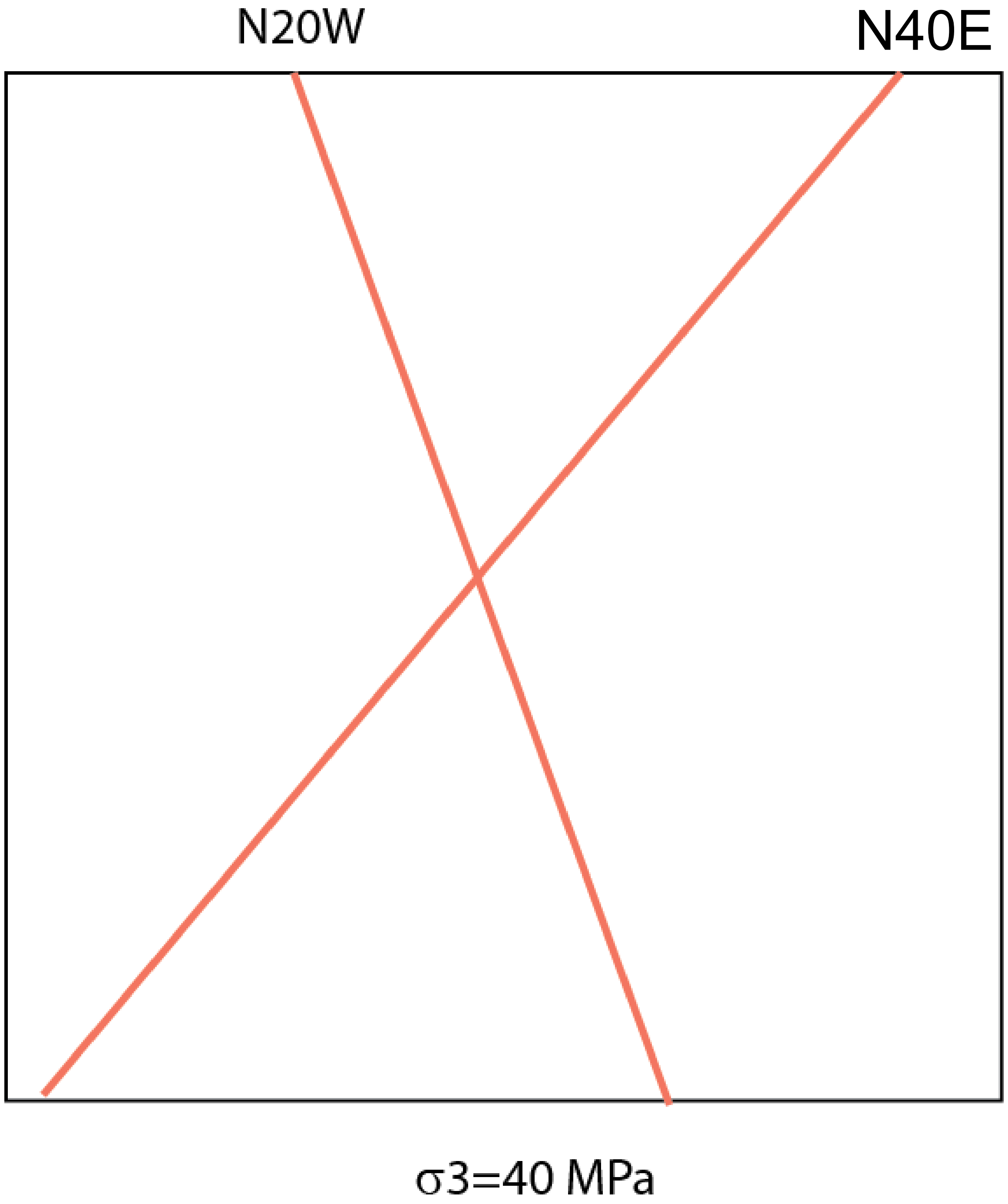
$$\sigma_s \quad \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

$$\frac{40 - 20}{2} \sin 2 \cdot 34$$

$$\frac{20}{2} \sin 68$$

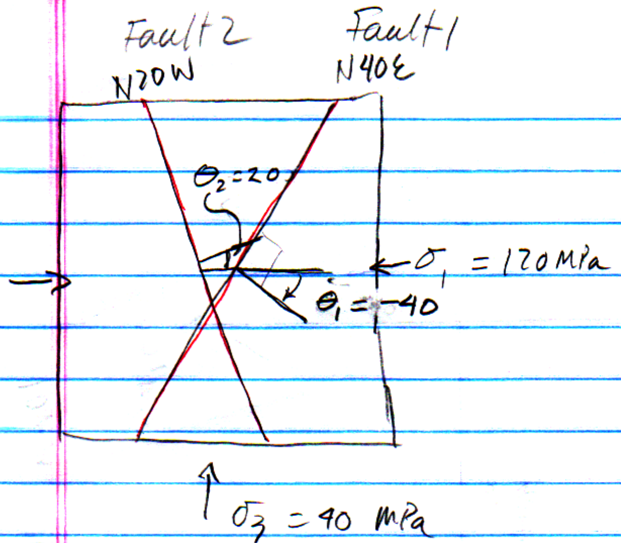
$$10 \cdot 0.927 = 9.3 \text{ MPa}$$





Using both the Mohr circle *and* the fundamental stress equations, determine the normal and shear tractions on the two planes. For Mohr circle, use 1 cm = 10 MPa)

$\sigma 1 = 120$ MPa



Normal traction

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

Shear traction

$$\sigma_s = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

F_1

$$= \frac{120 + 40}{2} + \frac{120 - 40}{2} \cos 2 \cdot -40$$

$$= \frac{160}{2} + \frac{80}{2} \cos(-80)$$

$$= 80 + 40 \cdot 0.17$$

$$80 + 6.9$$

$$= 87 \text{ MPa}$$

$$= \frac{120 - 40}{2} \sin 2 \cdot -40$$

$$= \frac{80}{2} \sin -80$$

$$= 40 \cdot \sin -80$$

$$= -39.4 \text{ MPa}$$

F_2

$$= \frac{120 + 40}{2} + \frac{120 - 40}{2} \cos 2 \cdot 20$$

$$= 80 \quad 40 \quad \cos 40$$

$$= 80 \quad 30.6$$

$$= 110.6 \text{ MPa}$$

$$= \frac{120 - 40}{2} \sin 2 \cdot 20$$

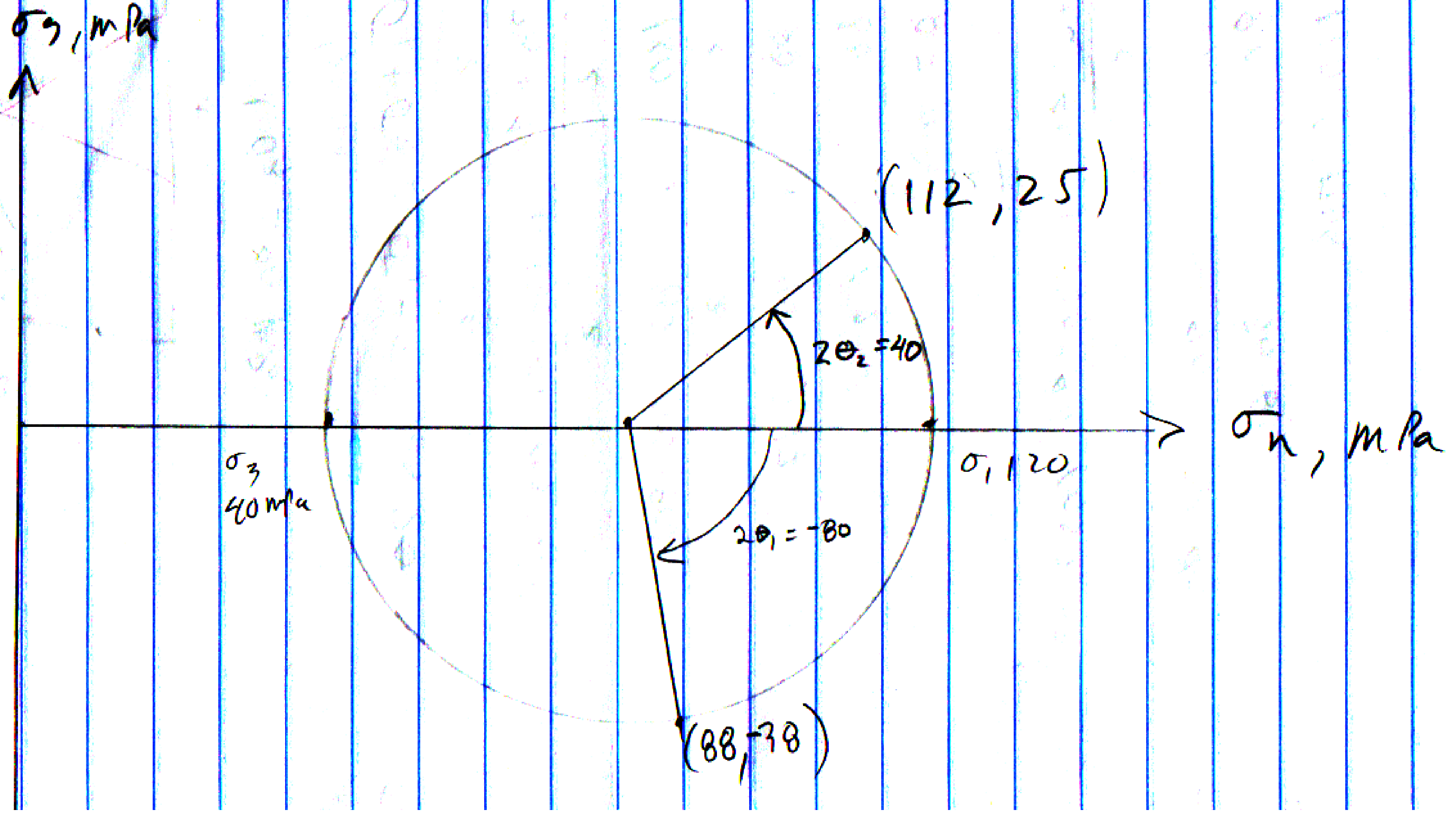
$$= \frac{80}{2} \sin 40$$

$$= 40 \cdot 0.64$$

$$= 25.7 \text{ MPa}$$

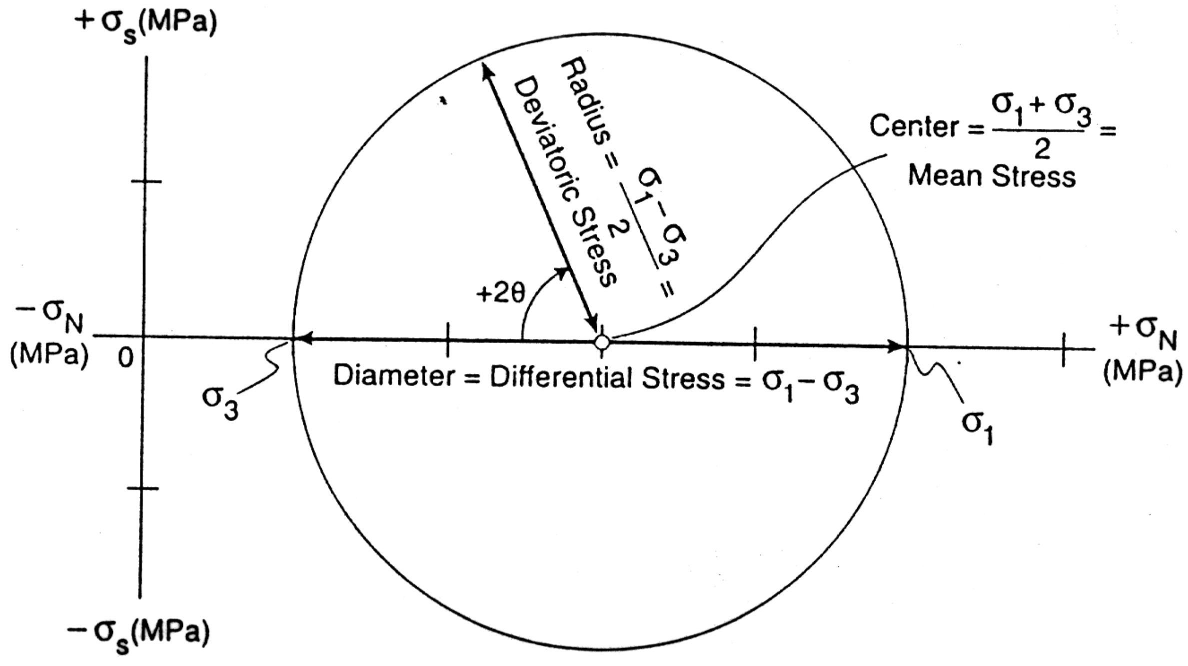
Mohr Circle

10mm = 10 mPa



Faults and stress: traction and stress

Figure 3.24 The center of the Mohr stress circle represents mean stress, which is the hydrostatic component of the stress field. Mean stress tends to produce dilation. The radius of the Mohr stress circle represents deviatoric stress, which is the nonhydrostatic component of the stress field. Deviatoric stress tends to produce distortion. The diameter of the Mohr stress circle represents differential stress. The larger it is, the greater the potential for distortion.



-Davis and Reynolds

$\tau = \sigma_s$

-Engelder, 1993, Stress regimes in the lithosphere, Princeton Univ. Press

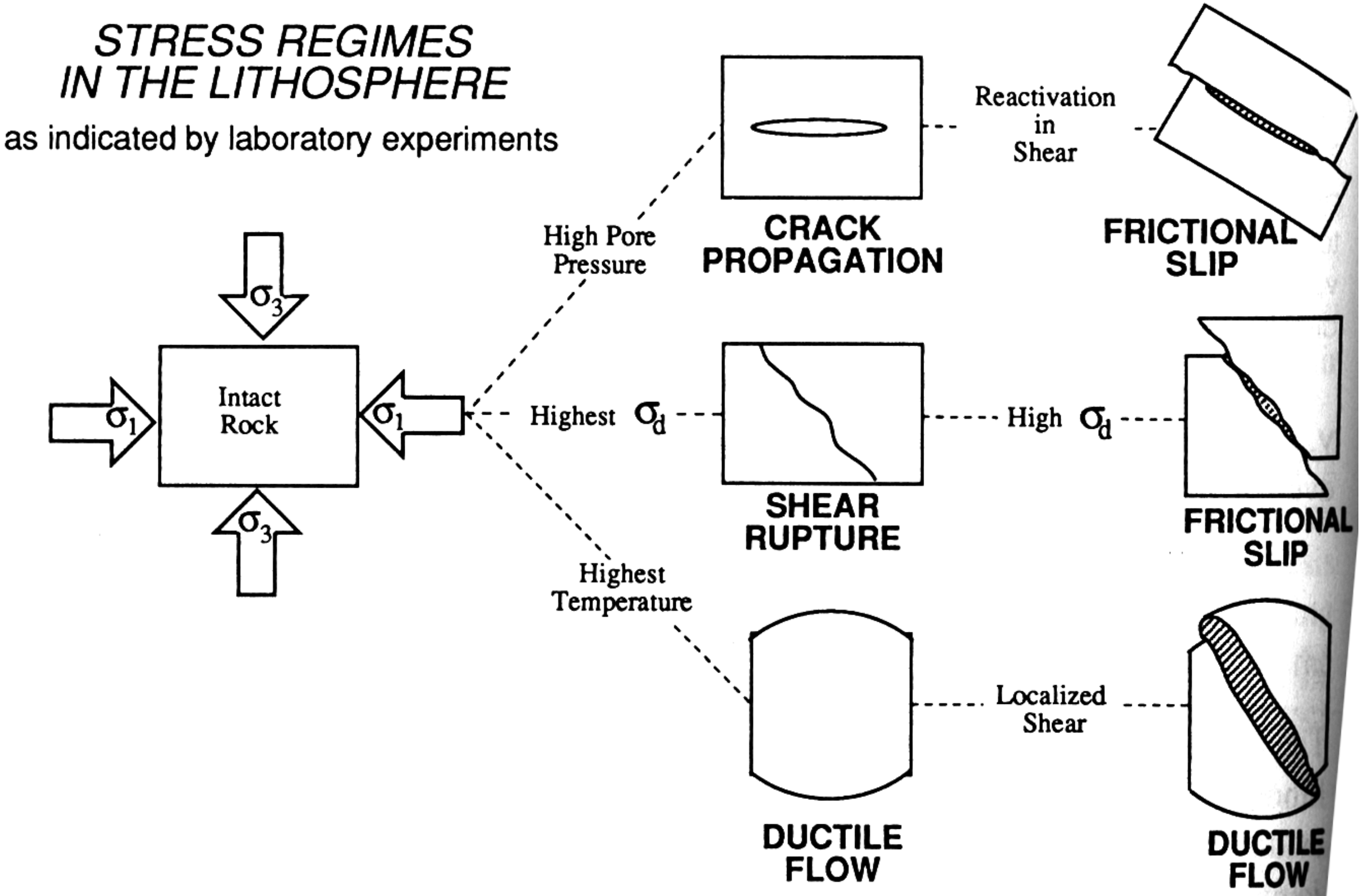
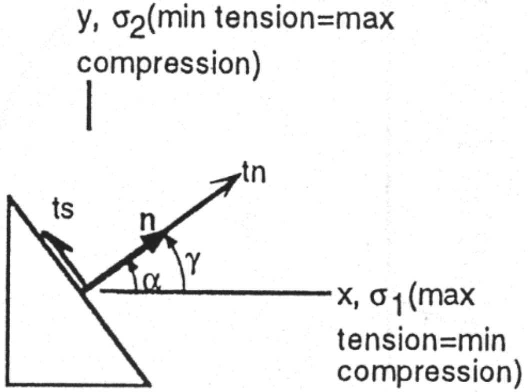


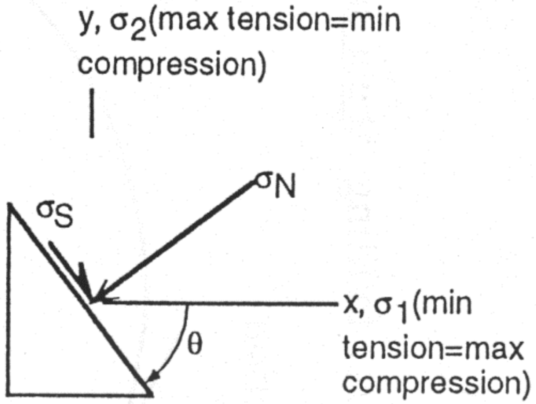
Fig. 1–6. The definition of stress regimes in the lithosphere based on the general types of failure encountered during laboratory rock mechanics experiments. Each rectangular box represents the cross section of a cylindrical sample during a polyaxial rock deformation experiment. In the laboratory, intact rock fails by three general mechanisms: crack propagation; shear rupture; and ductile flow. When subject to large shear stresses, joints and shear fractures are reactivated by frictional slip. Shear zones develop if ductile flow is localized. The four stress regimes of the lithosphere are identified by bold letters.

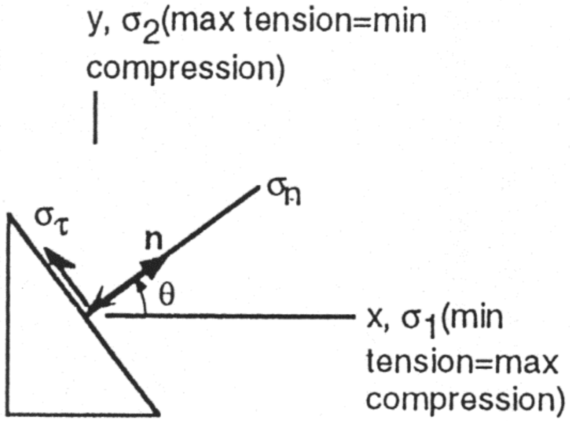
Mohr circle for stress--three ways to do it

Feature	Johnson and Pollard	Davis and Reynolds	Suppe, Ragan (most common)
Signs	Tension positive (σ_1 is max tension)	Compression positive (σ_1 is max compression)	Compression positive (σ_1 is max compression)
Axes	ts positive downward tn tension to the right	ts = σ_s positive upward tn = σ_n positive right	ts = σ_τ postivie upward tn = σ_n positive right
Angle-physical space	γ measured between σ_1 and **n** (because it is a principal plane, **t**(**n**) = tn, and ts = 0)	θ measured between σ_1 and **boundary**	θ measured between σ_1 and **n**
Angle-Mohr space	2γ measured **counterclockwise** from positive tn direction	2θ measured **clockwise** from negative σ_n direction	2θ measured **counterclockwise** from positive σ_n direction

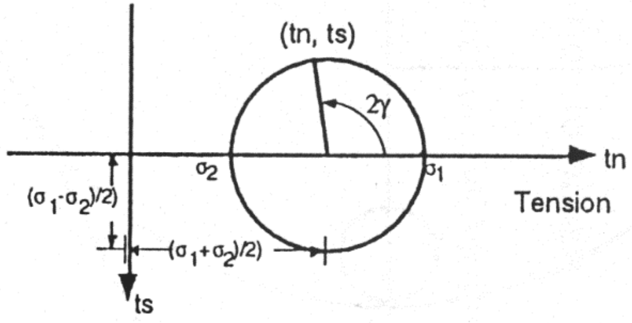
Geometry-physical space

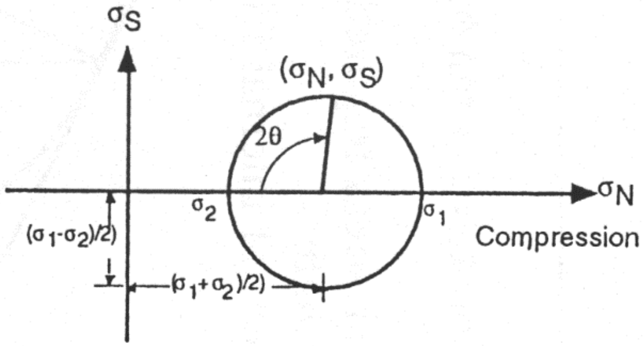


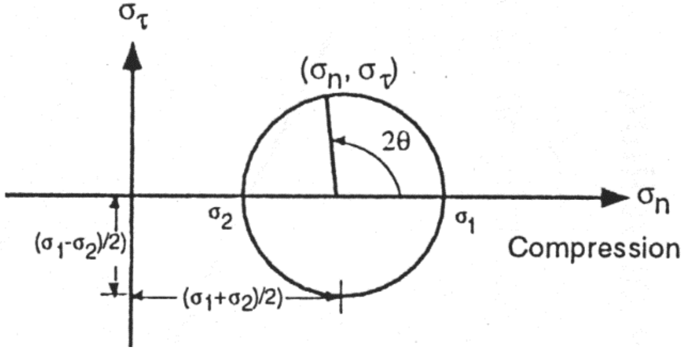




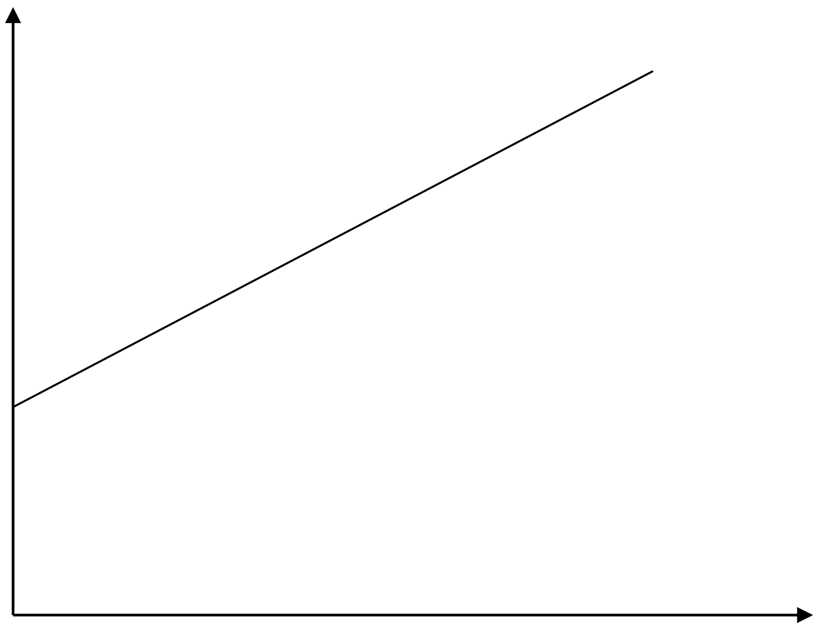
Geometry-Mohr space







Coulomb Law of Failure



Coulomb equation

$\tau_c = c + \tan\phi \, \sigma_n$

Where

τ_c = critical shear stress required for faulting (shear strength)

c = cohesive strength

$\tan\phi$ = coefficient of internal friction = μ

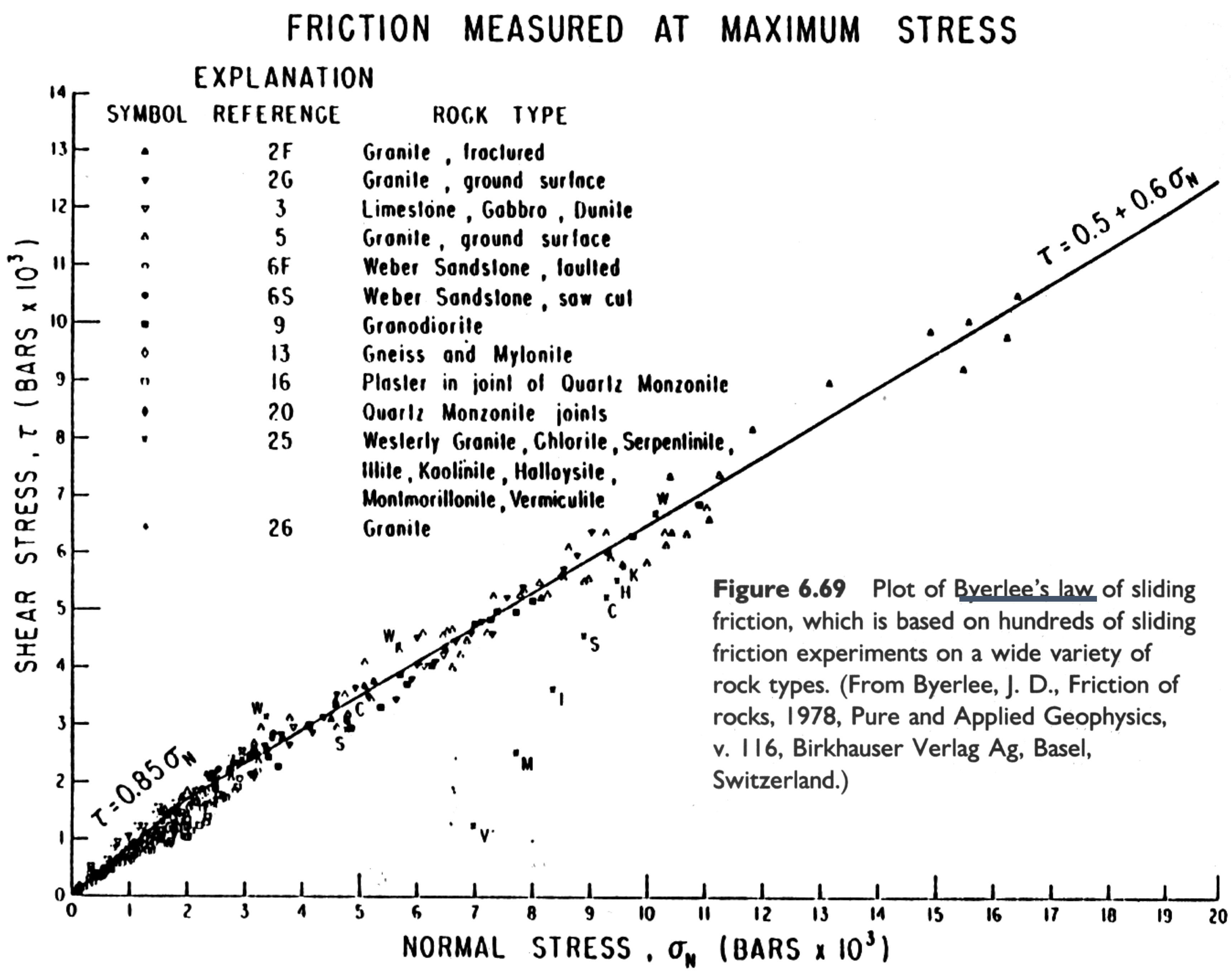
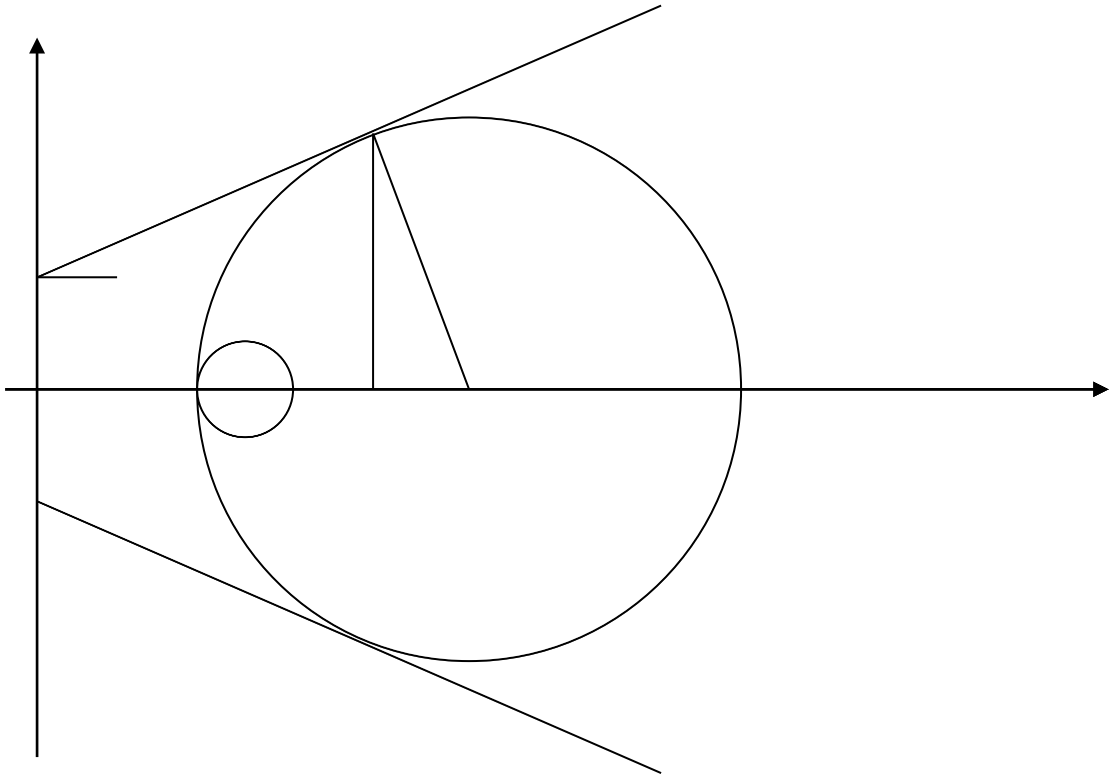
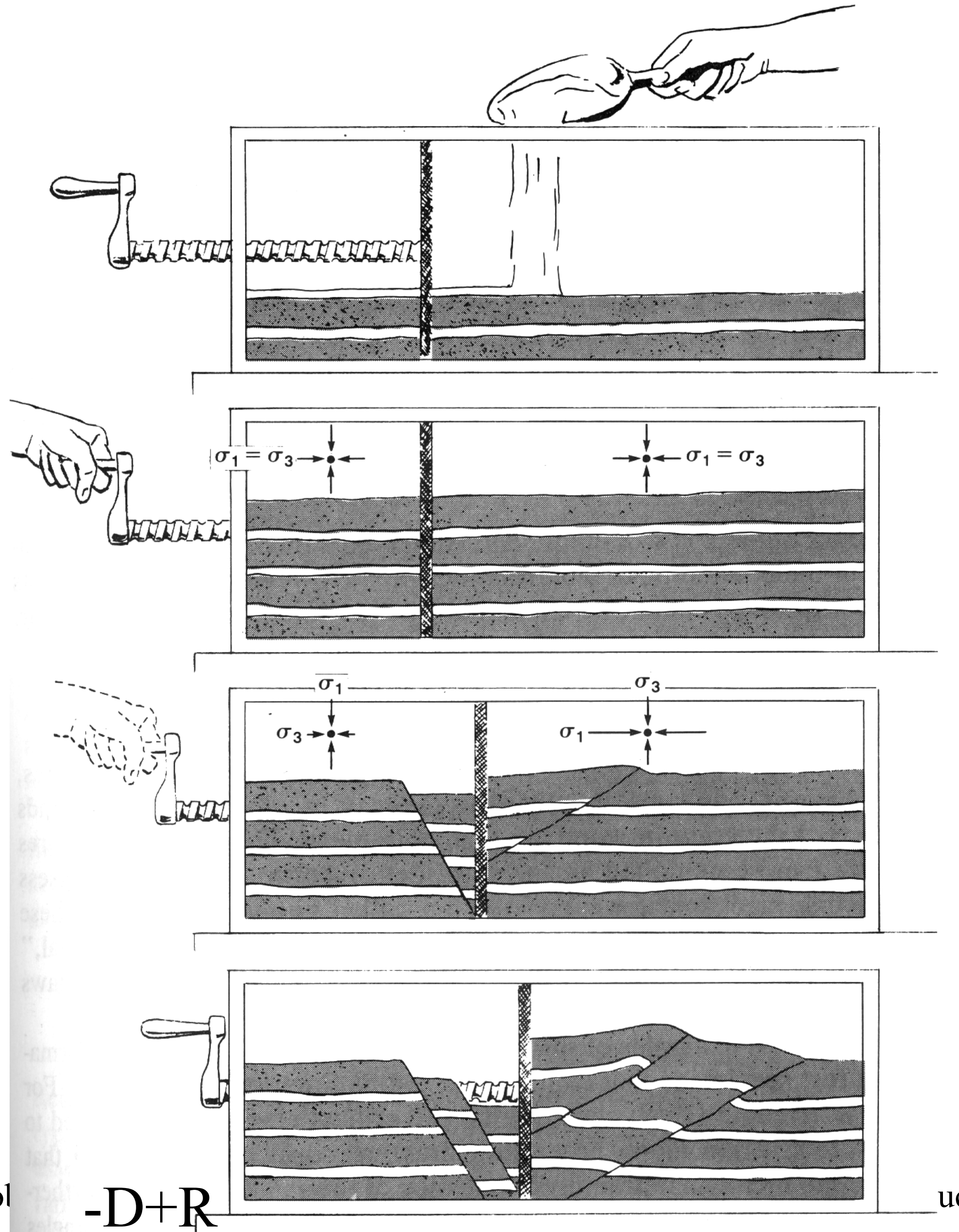


Figure 6.69 Plot of Byerlee's law of sliding friction, which is based on hundreds of sliding friction experiments on a wide variety of rock types. (From Byerlee, J. D., Friction of rocks, 1978, Pure and Applied Geophysics, v. 116, Birkhauser Verlag Ag, Basel, Switzerland.)

Faults and stress: traction and stress





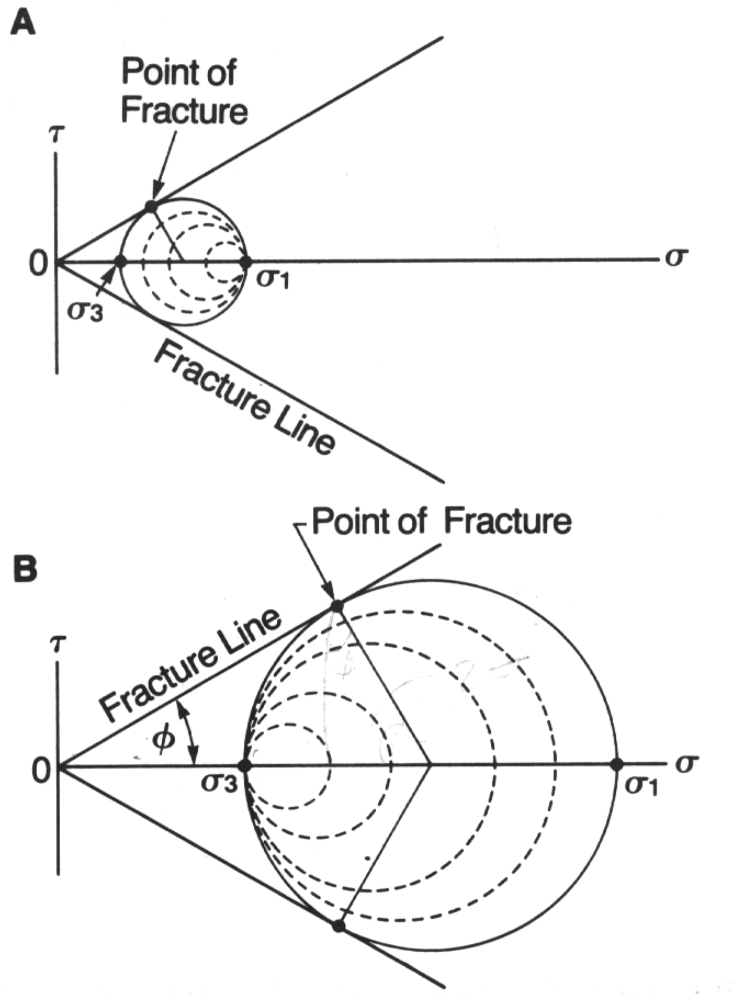
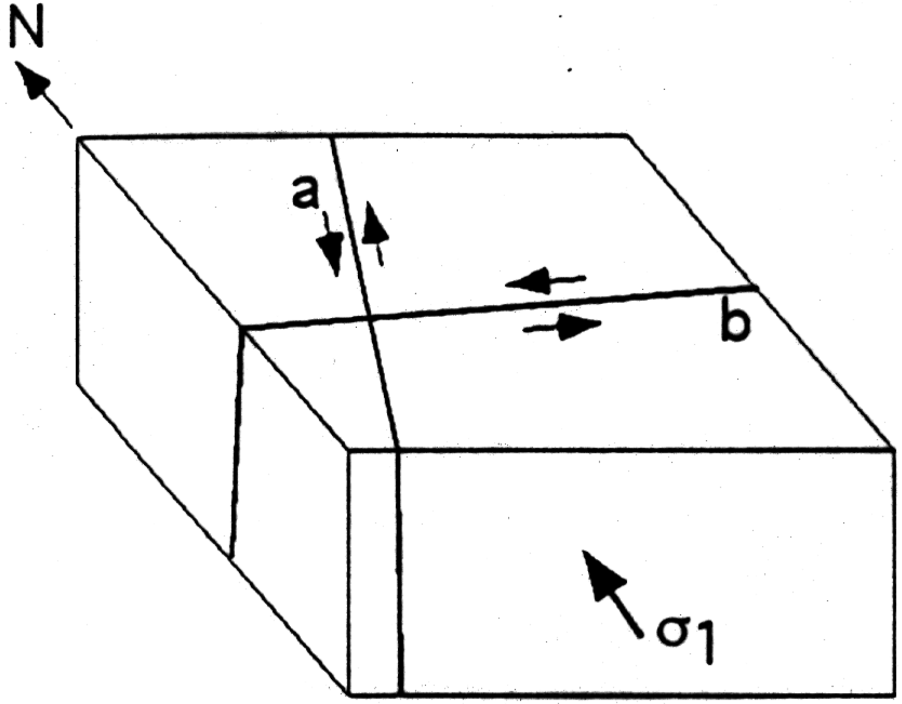


Figure 6.67 Mohr diagram portrayal of the dynamic conditions of the sandbox experiment. (*A*) Differential stress conditions leading to normal faulting in the left-hand compartment. (*B*) Differential stress conditions leading to thrust faulting in the right-hand compartment.

In the block below, σ_1 is oriented north-south. The area is in a regime of strike-slip faulting. $\sigma_1 = 100$ MPa and $\sigma_3 = 50$ Mpa, fault **a** strikes 015, and fault **b** strikes 085.



a) What are the normal and shear tractions acting along the two faults?

b) Which is more likely to fail?

c) If you assume that Byerlees's law ($\tau = 0.5 + 0.6\ \sigma_n$) holds, what would the strike of the optimally oriented fault be?