Advanced Structural Geology, Fall 2022

# Elastic Ductile Deformation

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Content from Structural Geology: A Quantitative Introduction by Pollard and Martel

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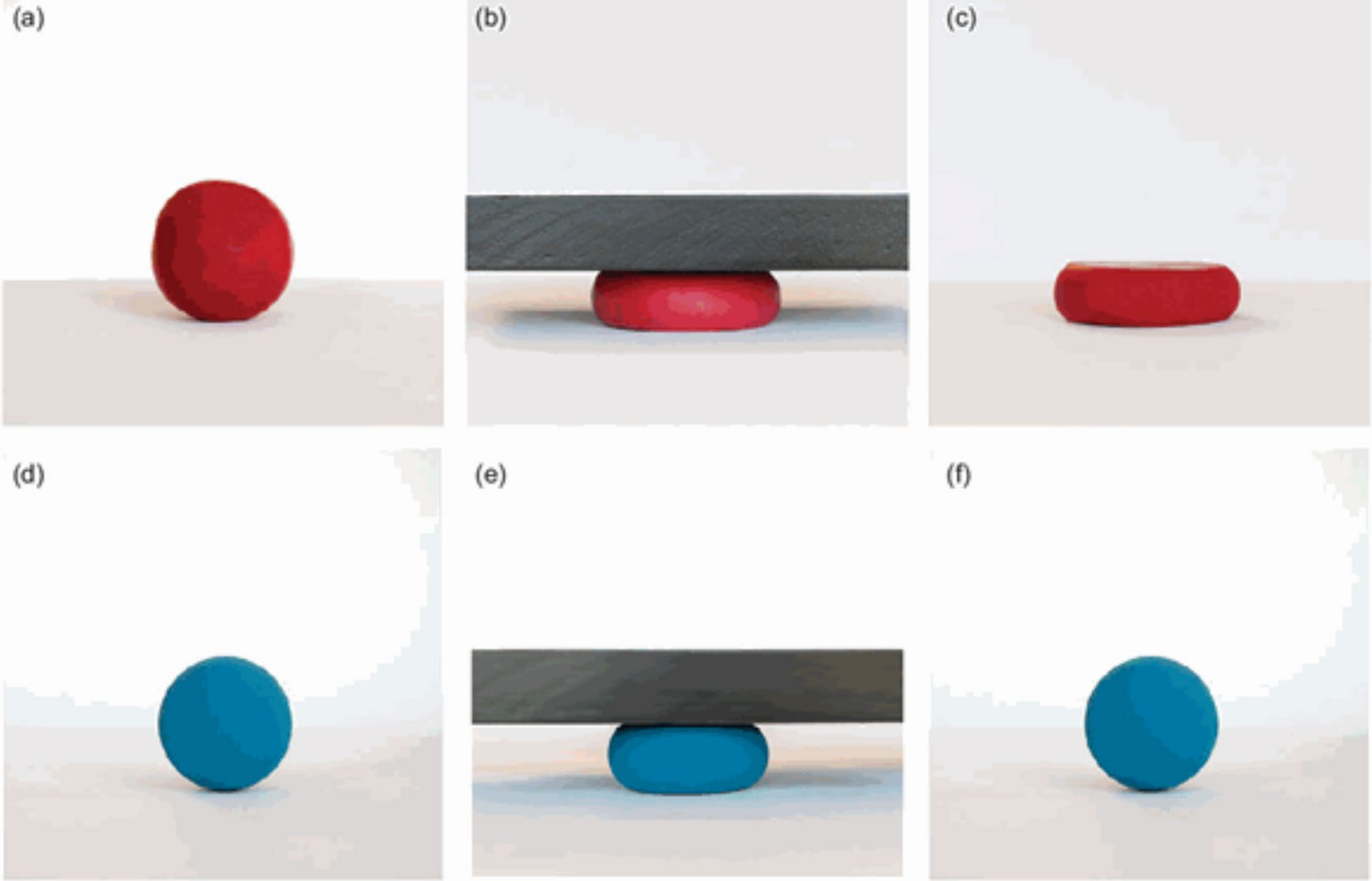
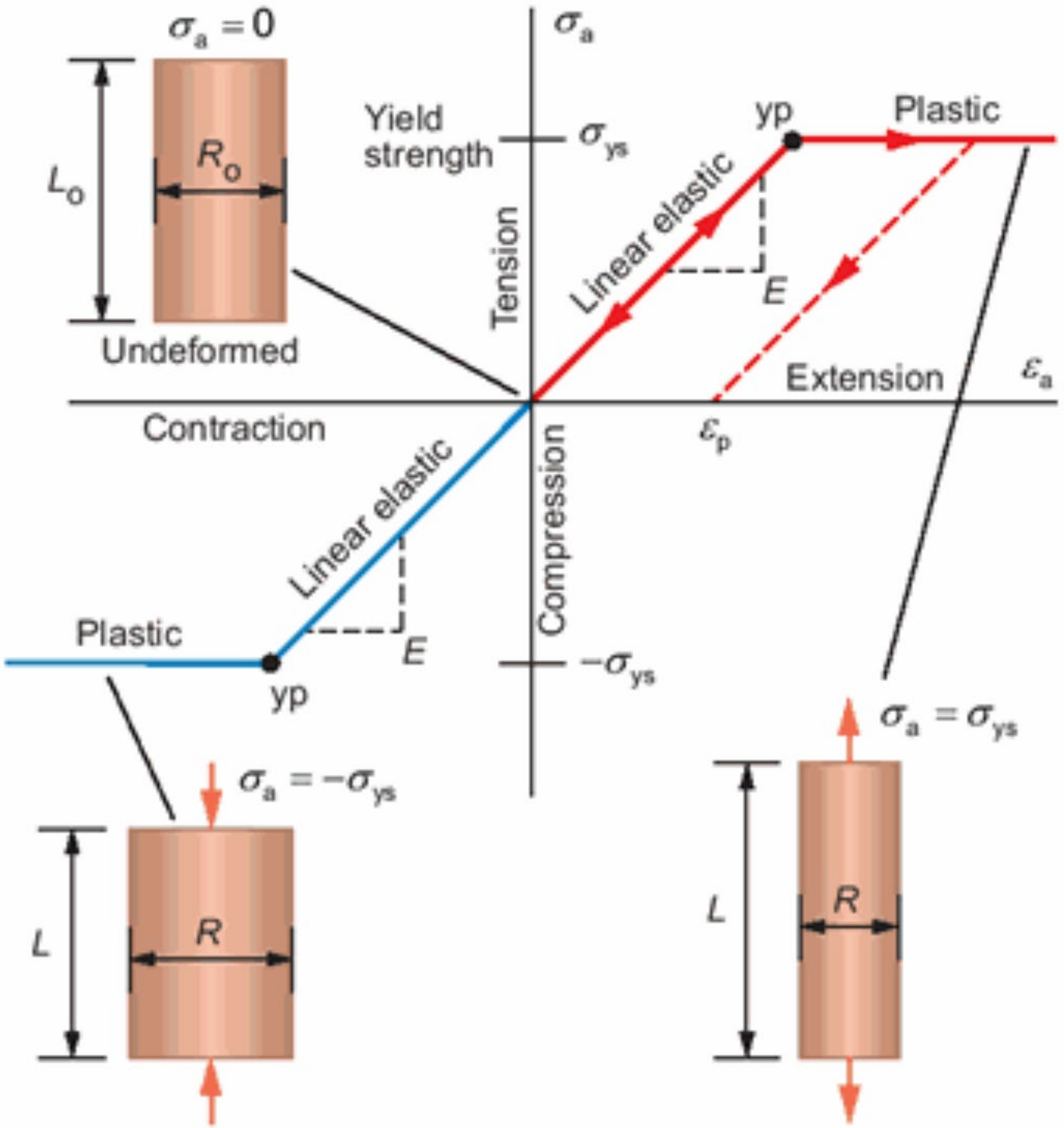


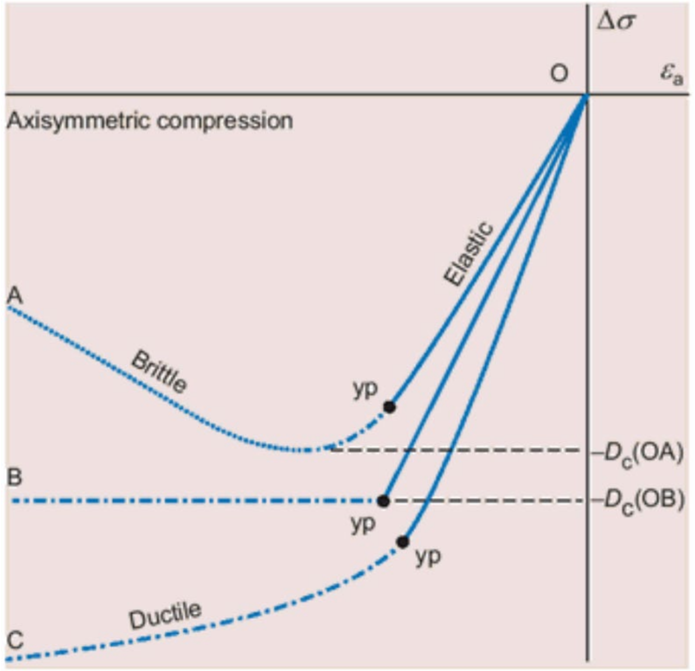

Figure 5.1 Three stages in the deformation of a modeling clay sphere (red) and a rubber racquetball (blue). (a) Unloaded sphere rests on tabletop. (b) Weight (gray) imposes stress equal to the yield strength of the clay and it deforms to a flattened disk. (c) Weight removed; clay remains a flattened disk. (d) Unloaded rubber ball rests on tabletop. (e) Weight (gray) imposes stress and ball flattens. (f) Weight removed; ball springs back to its original shape. Photography by Richard Stultz.

Plastic: applied stress equal to the yield strength produces distributed and Irrecoverable strain

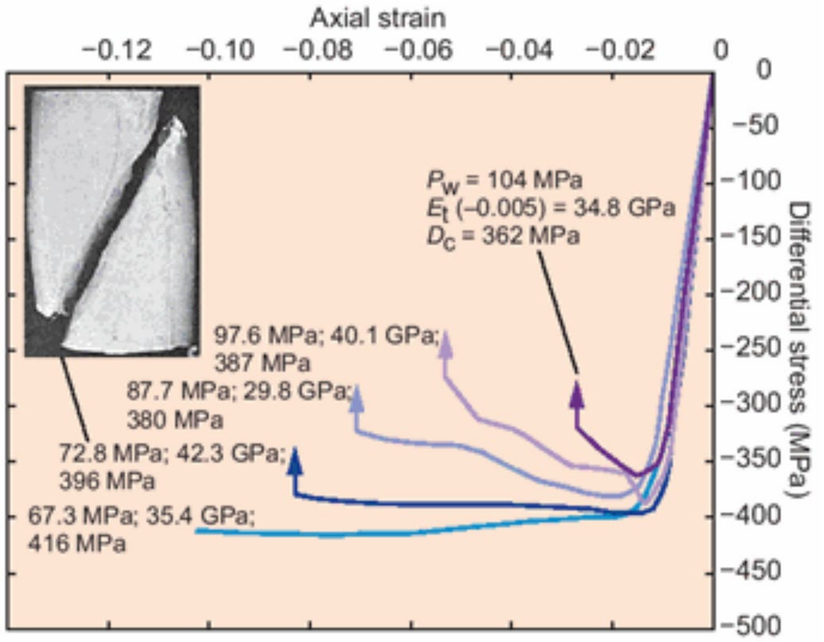
Elastic: applied stress produces distributed and recoverable strain



σa = 0
Lo
Ro
Undeformed
σa
Yield strength
σys
Tension
yp
Plastic
Linear elastic
E
Extension
εa
Contraction
εp
Compression
Linear elastic
E
Plastic
yp
−σys
σa = σys
σa = −σys
L
R
L
R

Δσ
O
εa
Axisymmetric compression
Elastic
A
Brittle
yp
−Dc(OA)
B
−Dc(OB)
yp
yp
Ductile
C

Axial strain
−0.12
−0.10
−0.08
−0.06
−0.04
−0.02
0
Differential stress (MPa)
0
−50
−100
−150
−200
−250
−300
−350
−400
−450
−500
Pw = 104 MPa
Et (−0.005) = 34.8 GPa
Dc = 362 MPa
97.6 MPa; 40.1 GPa; 387 MPa
87.7 MPa; 29.8 GPa; 380 MPa
72.8 MPa; 42.3 GPa; 396 MPa
67.3 MPa; 35.4 GPa; 416 MPa

# Outcrop Scale

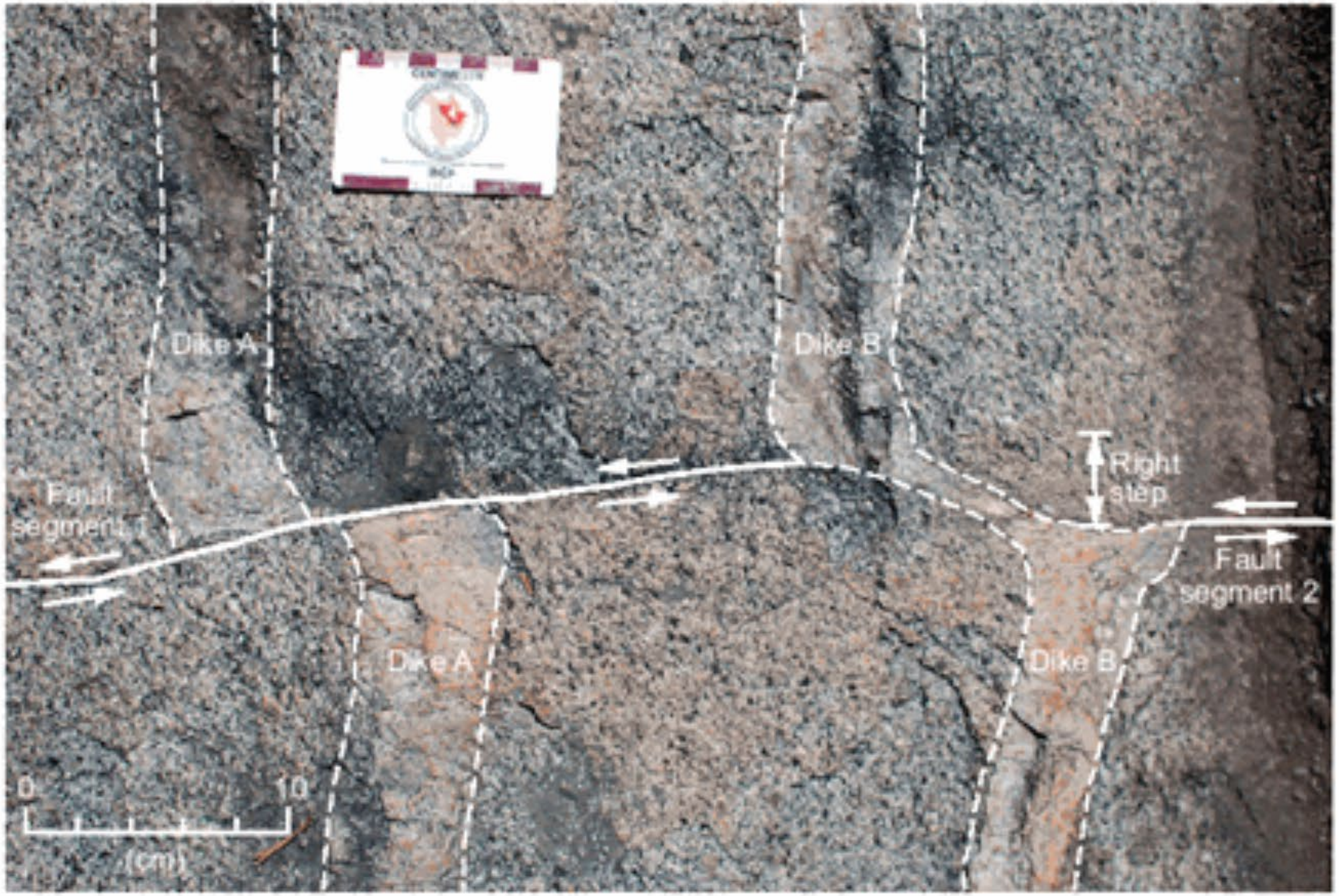
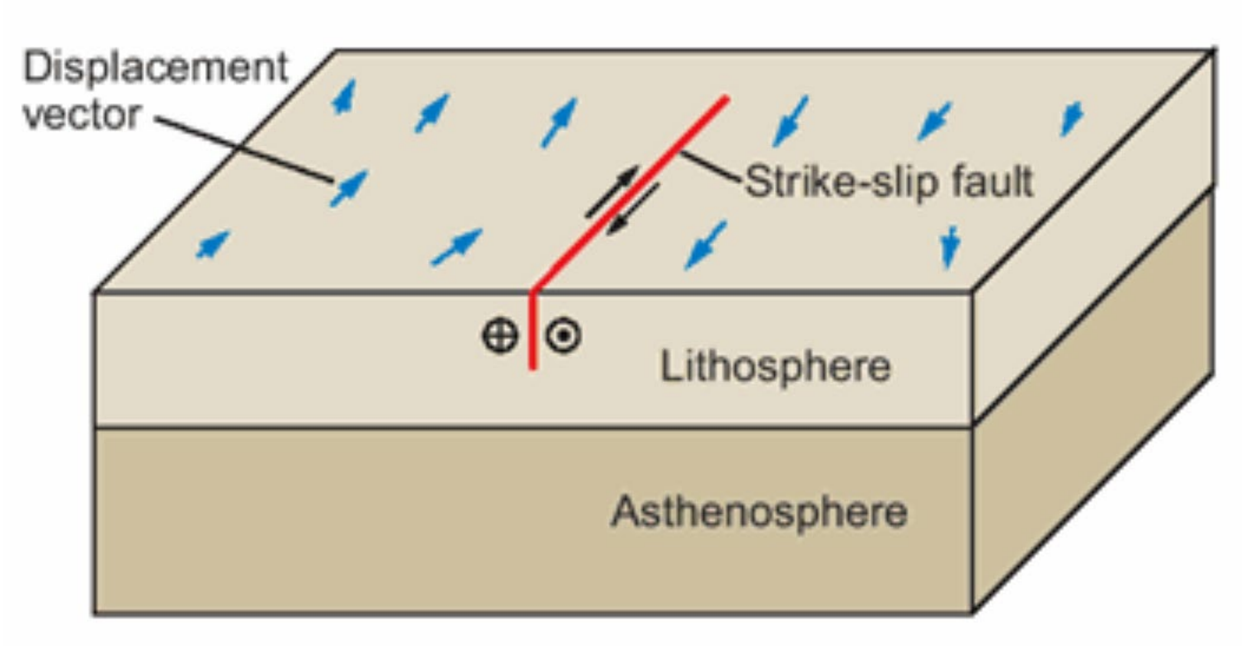
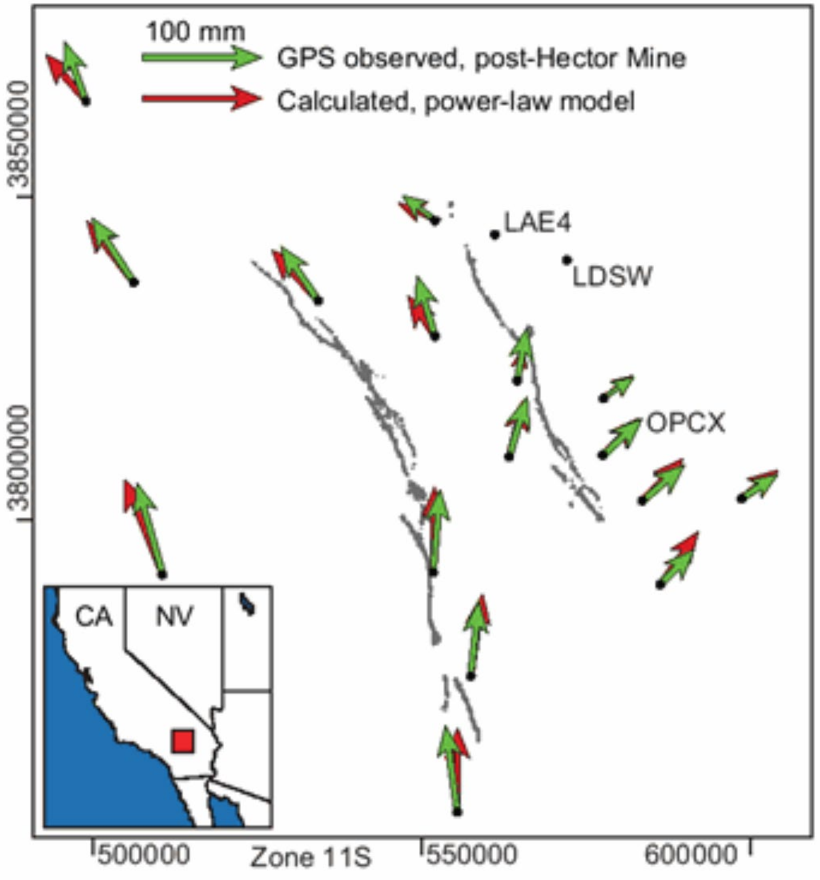



Figure 5.3 Two echelon segments of a left-lateral fault in Lake Edison Granodiorite, Sierra Nevada, CA with two offset dikes. Dike A is broken and offset about 6 cm along fault segment 1, but little deformed. Dike B is offset about 11 cm and is rotated, stretched and thinned in the right step between fault segments 1 and 2. Photograph by J. M. Nevitt.

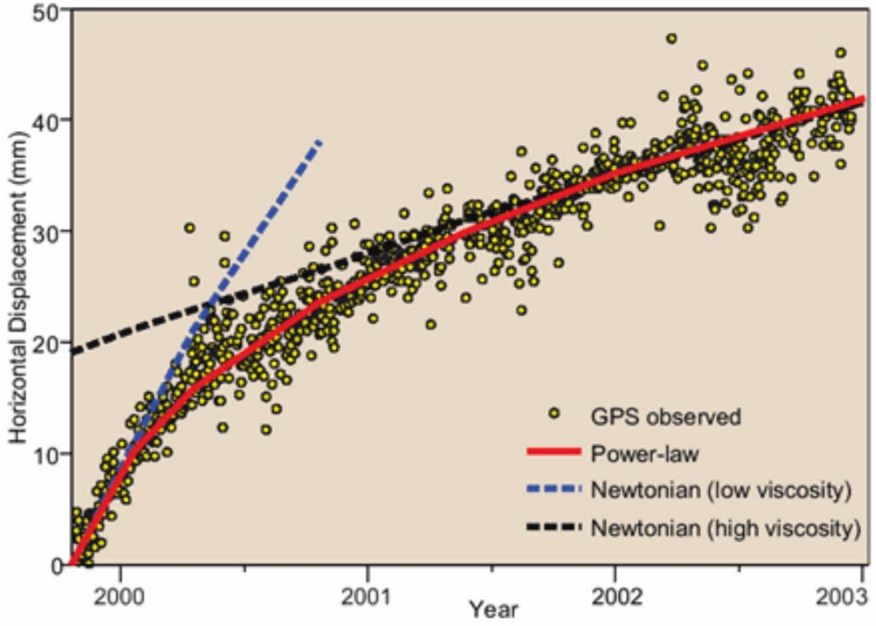
# Crustal Scale

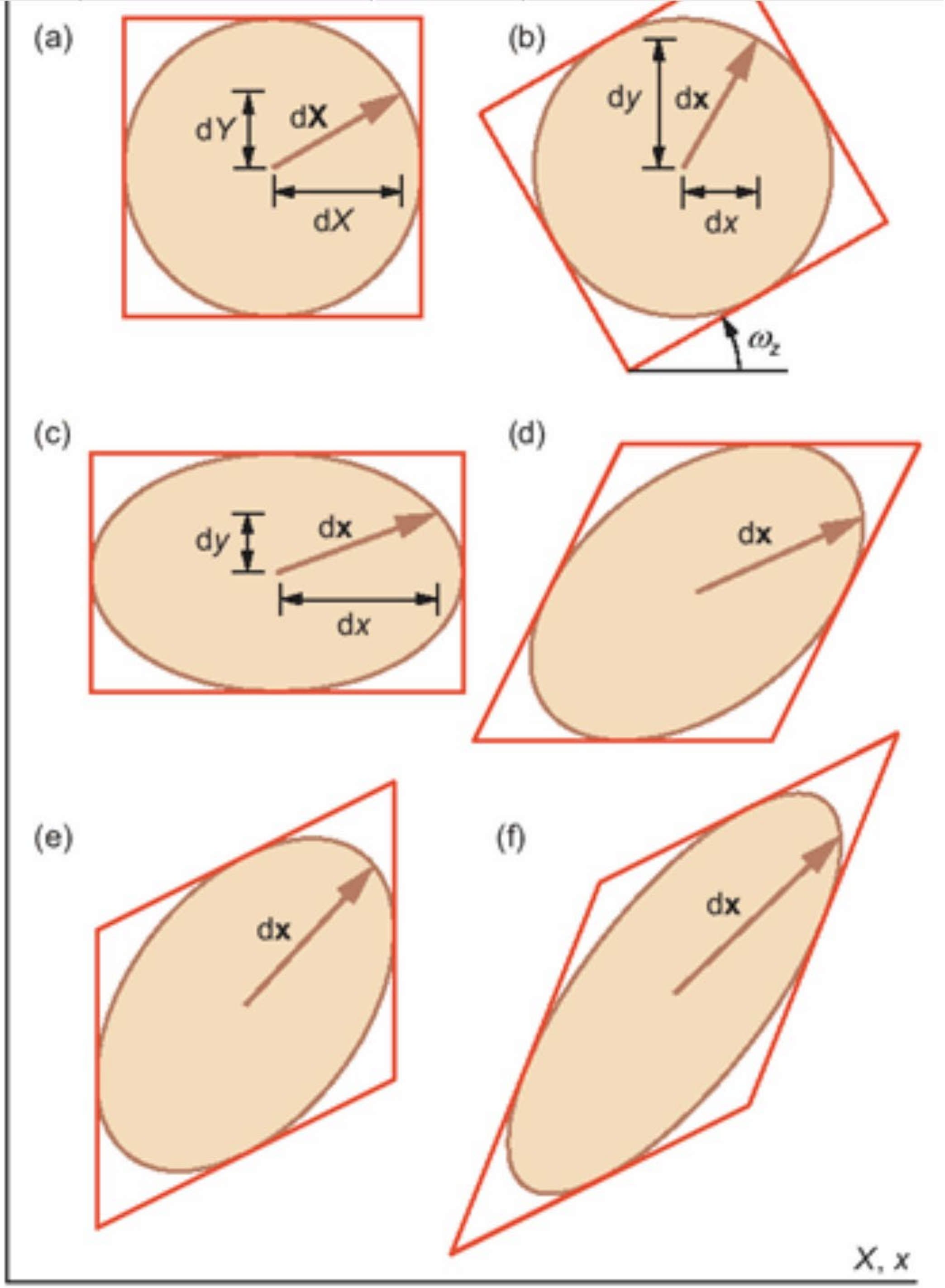



$$\dot{\varepsilon} = A(\Delta\sigma)^n \exp(-Q/RT)$$

Flow law:
A constant
Edot axial strain
$\Delta\sigma$ Differential stress
Q activation energy
R universal gas constant
T temperature
n power law exponent



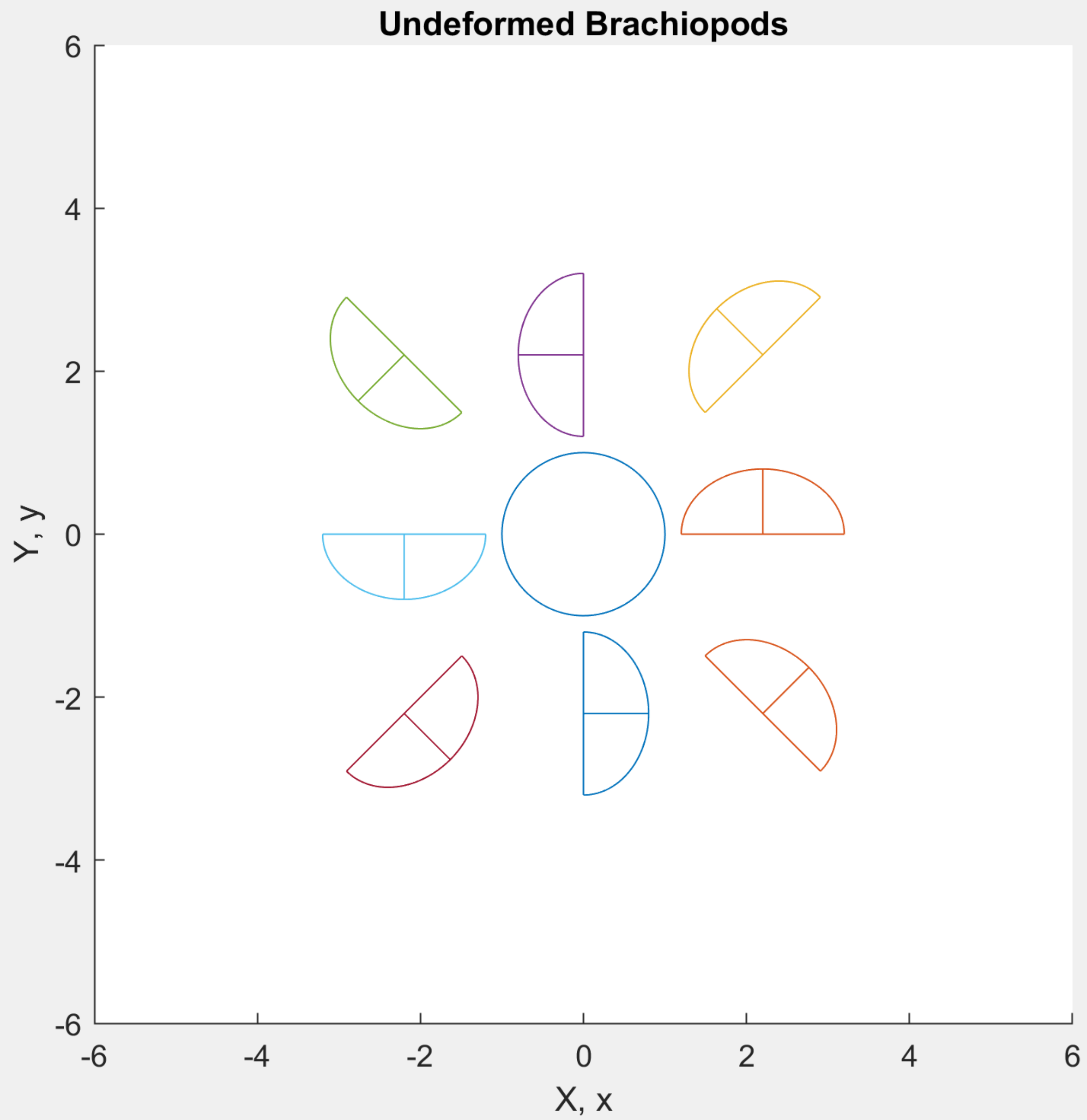


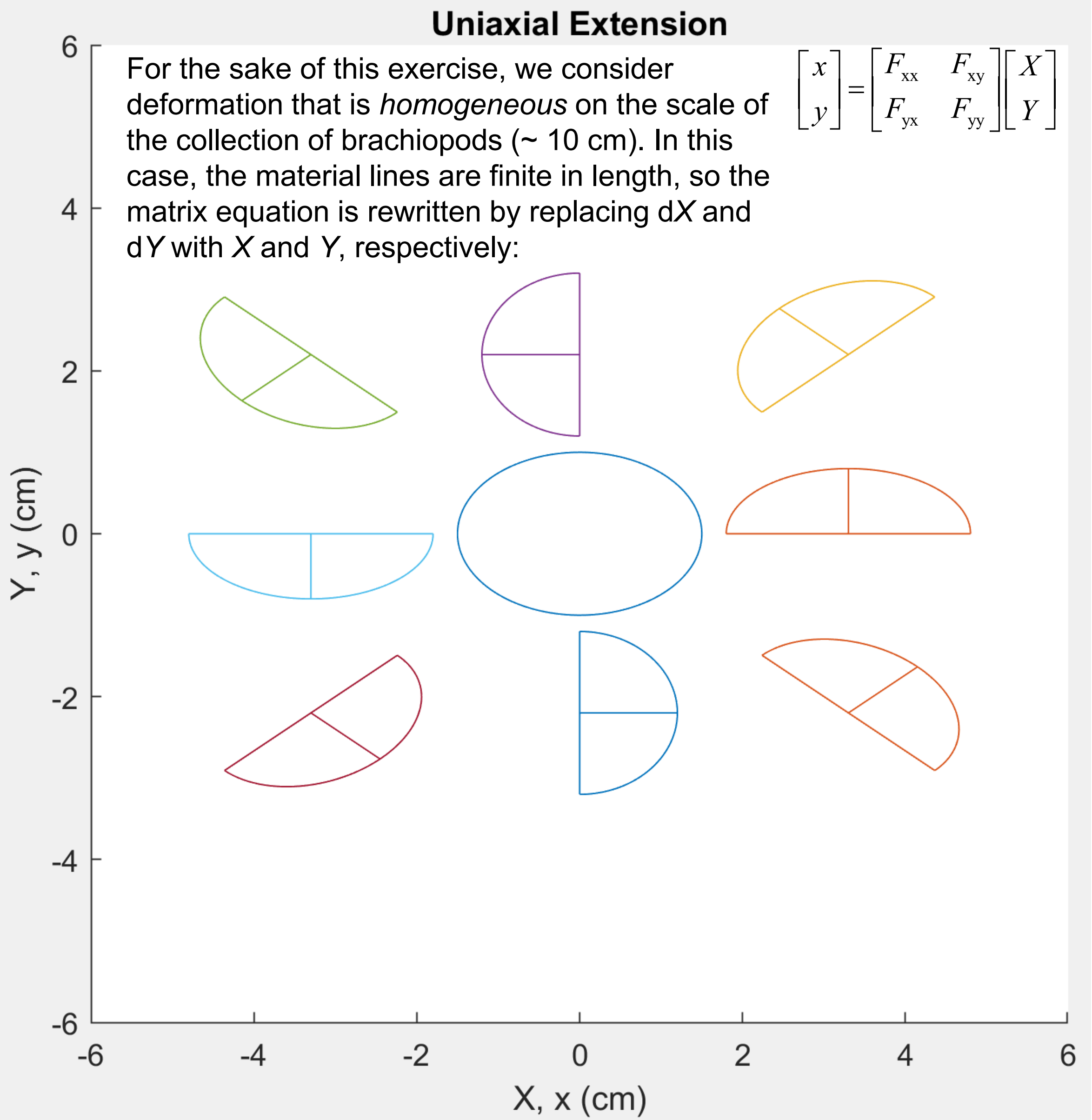

Rotation

Pure shear and simple shear in x

Simple shear in y and simple shear in x and y

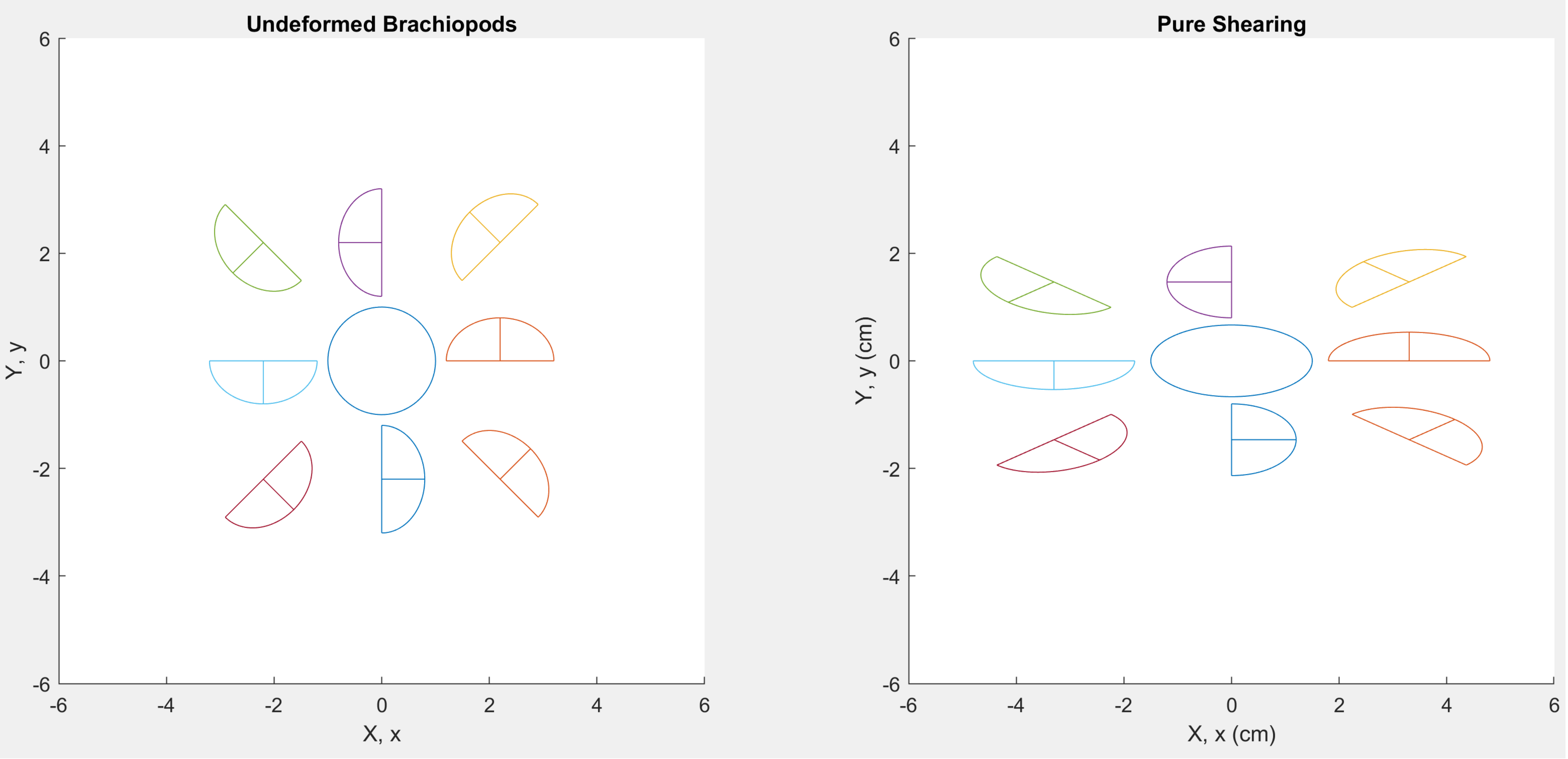
```
% Ch5_Q3.m
% deform idealized brachiopods and oolith
% using a linear transformation for homogeneous
deformation in 2D
clearvars; close all; % clear workspace variables,
close figures

figure(1)
clf
subplot(1,2,1)
% plot a circle (future stretch ellipse)
the=(0:1:360)*pi/180;
xe=cos(the);ye=sin(the);
hold on, axis equal
axis([-6 6 -6 6])
plot(xe,ye)

% compute generic brachiopod
b=0.8; % central plication/hinge line
thet=(0:1:90)*pi/180;
x2=cos(thet);y2=b*sin(thet);
thet=(180:-1:90)*pi/180;
x5=cos(thet);y5=b*sin(thet);
x=[0 x2 0 -1 x5];y=[0 y2 0 0 y5];

% position and plot undeformed brachiopods
xp=[2.2 2.2 0 -2.2 -2.2 -2.2 0 2.2];
yp=[0 2.2 2.2 2.2 0 -2.2 -2.2 -2.2];
oms=[0 45 90 135 180 225 270 315]*pi/180;
% oms=[2*pi*rand(size(xp))];
for ii=1:8
   om=oms(ii);
   com=cos(om); som=sin(om);
   xr=x*com-y*som+xp(ii);
   yr=x*som+y*com+yp(ii);
   plot(xr,yr)
end
title('Undeformed Brachiopods')
xlabel('X, x'), ylabel('Y, y')

subplot(1,2,2)
hold on, axis equal
% deform oolith and brachiopods
%Fxx=3/2;Fxy=0;Fyx=0;Fyy=1; % uniaxial extension
Fxx=3/2;Fxy=0;Fyx=0;Fyy=2/3; % biaxial extension (no vol change)
% Fxx=3/2;Fxy=0;Fyx=0;Fyy=3/2; % biaxial extension (pure dilation)
% Fxx=1;Fxy=1;Fyx=0;Fyy=1; % simple shear

axis([-6 6 -6 6])
xedef=Fxx*xe+Fxy*ye;
yedef=Fyx*xe+Fyy*ye;
plot(xedef,yedef) % plot deformed oolith

for ii=1:8
   om=oms(ii);
   com=cos(om); som=sin(om);
   xr=x*com-y*som+xp(ii);
   yr=x*som+y*com+yp(ii);
   xdef=Fxx*xr+Fxy*yr;
   ydef=Fyx*xr+Fyy*yr;
   plot(xdef,ydef) % plot deformed brachs
end
%title('Uniaxial Extension')
title('Pure Shearing')
% title('Pure Dilation')
% title('Simple Shearing')
xlabel('X, x (cm)'), ylabel('Y, y (cm)')
```



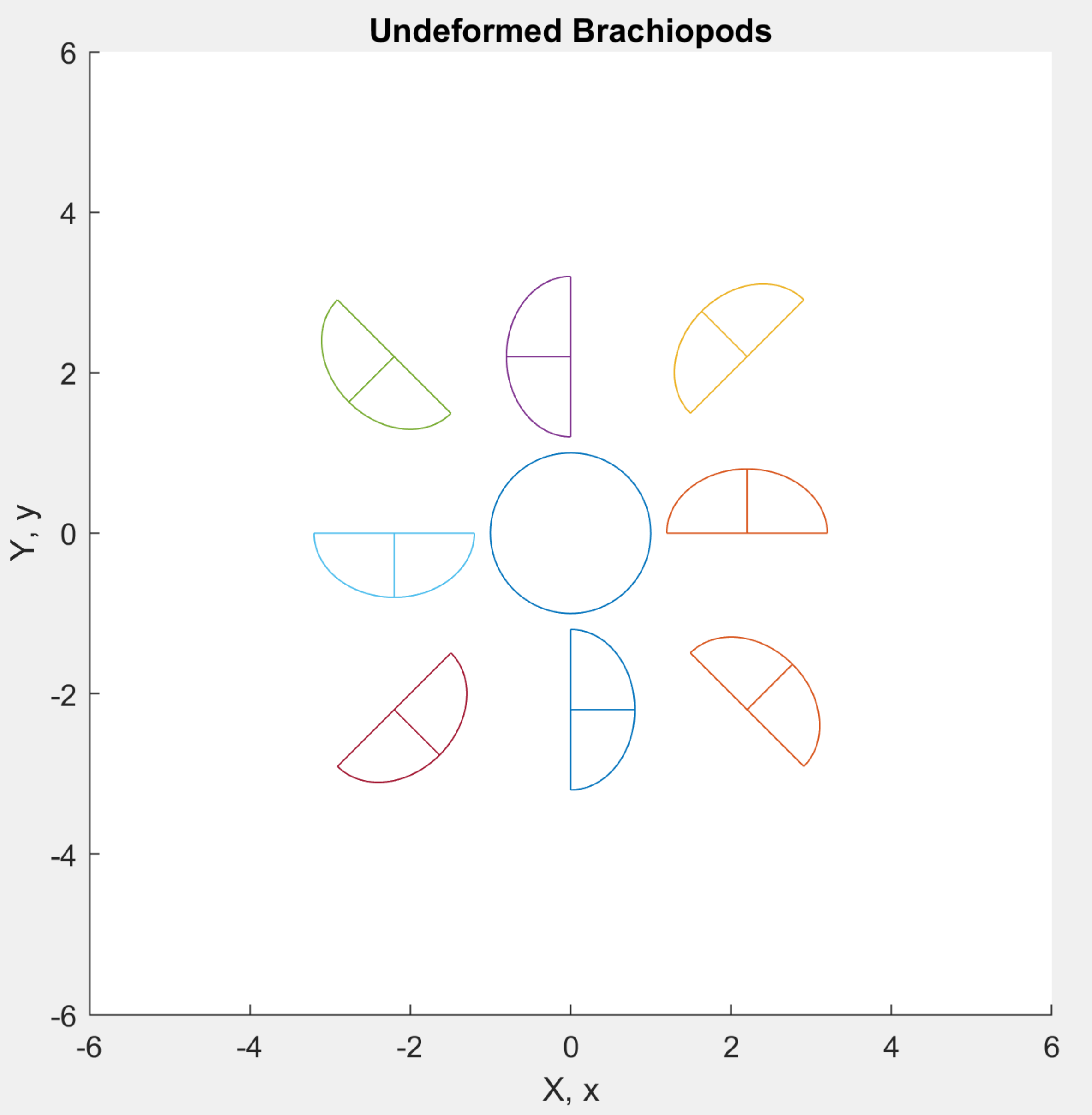

```
Fxx=3/2;Fxy=0;Fyx=0;Fyy=1; % uniaxial extension
```

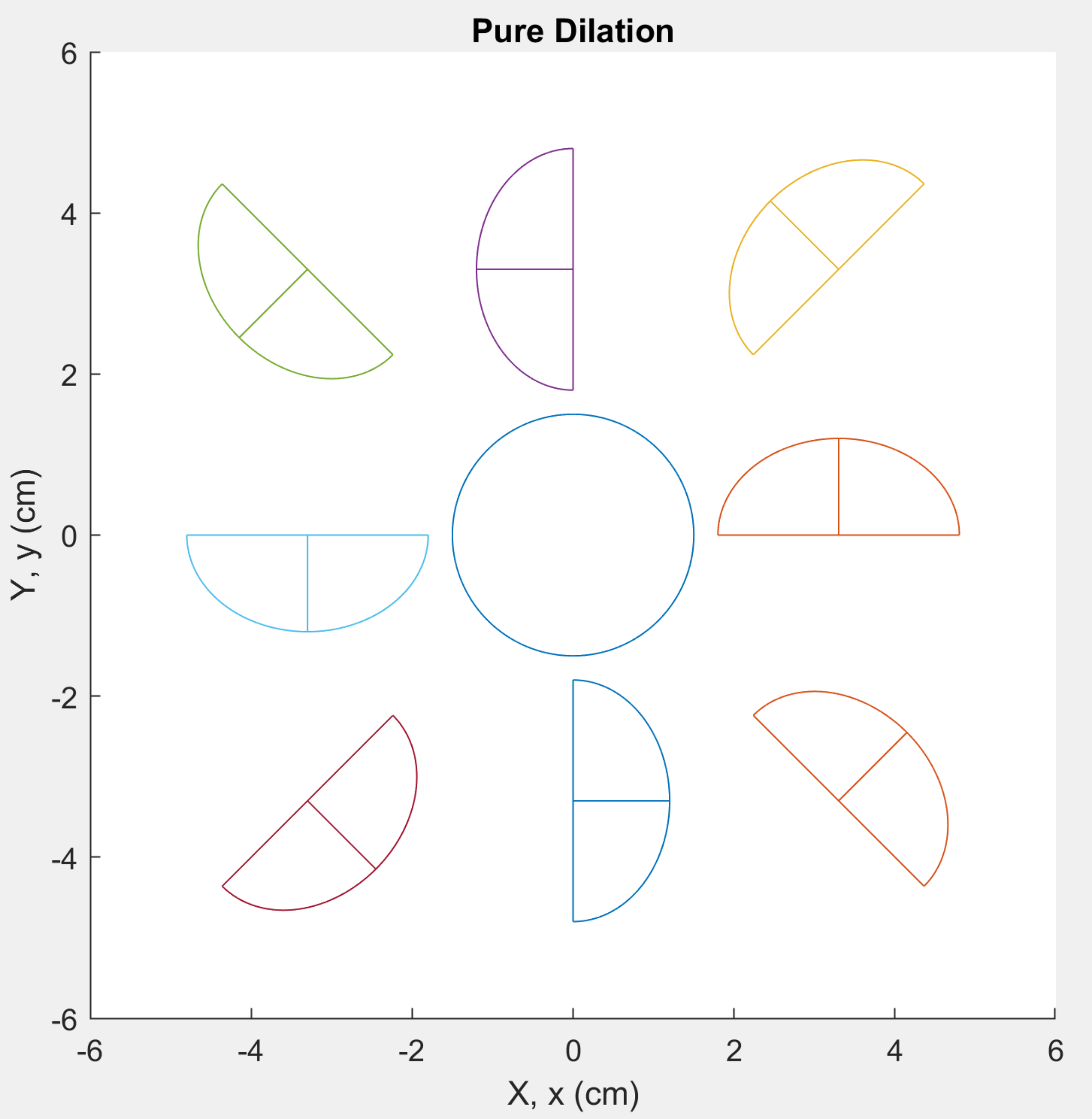
```
xdef=Fxx*xr+Fxy*yr;
ydef=Fyx*xr+Fyy*yr;
plot(xdef,ydef) % plot deformed brachs
```




```
Fxx=3/2;Fxy=0;Fyx=0;Fyy=2/3;
% biaxial extension (no vol change)
```

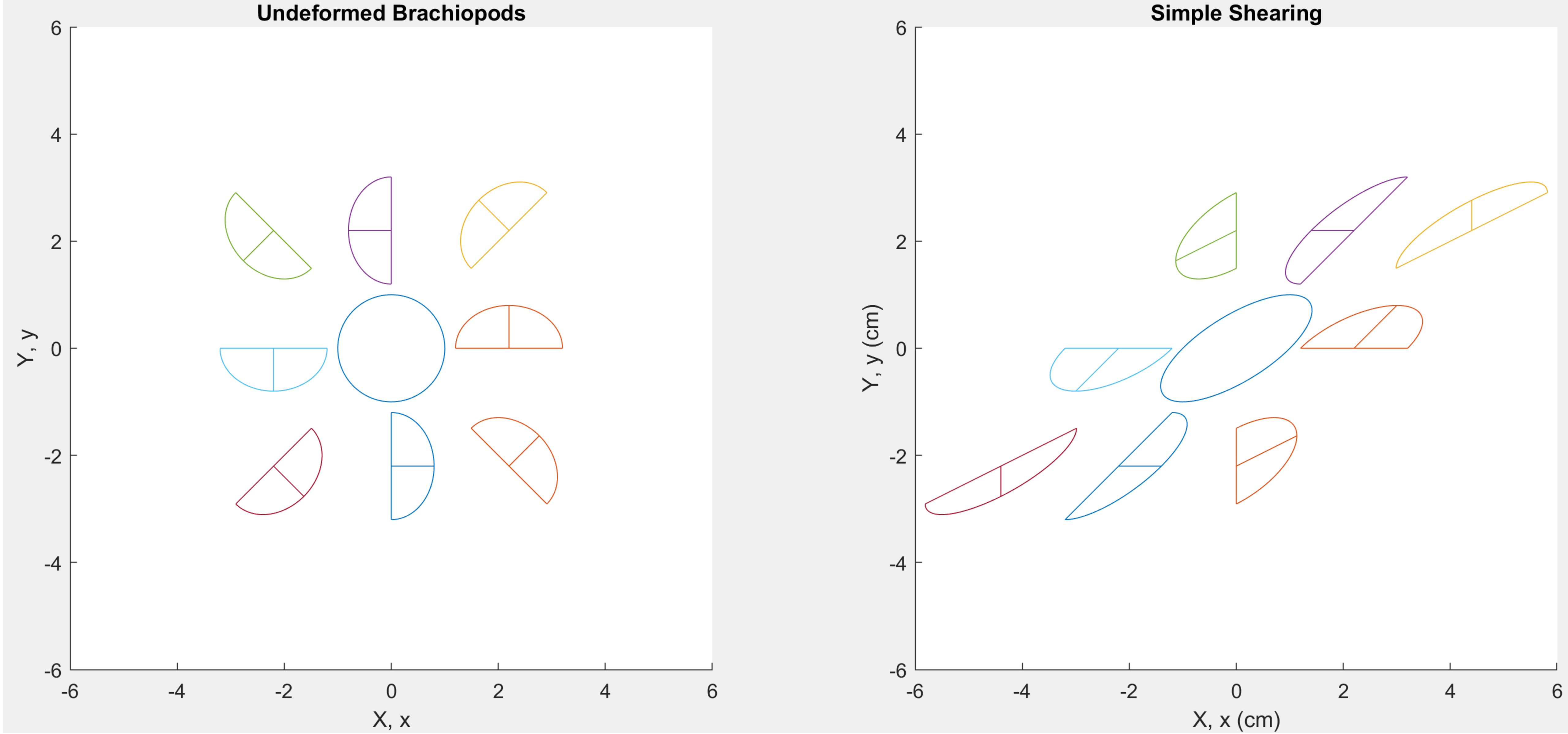
```
xdef=Fxx*xr+Fxy*yr;
ydef=Fyx*xr+Fyy*yr;
plot(xdef,ydef) % plot deformed brachs
```




```
Fxx=3/2;Fxy=0;Fyx=0;Fyy=3/2; %
biaxial extension (pure dilation)
```

```
xdef=Fxx*xr+Fxy*yr;
ydef=Fyx*xr+Fyy*yr;
plot(xdef,ydef) % plot deformed brachs
```




```
Fxx=1;Fxy=1;Fyx=0;Fyy=1; % simple shear
```

```
xdef=Fxx*xr+Fxy*yr;
ydef=Fyx*xr+Fyy*yr;
plot(xdef,ydef) % plot deformed brachs
```

## Deformation Gradient Tensor

$$[\mathrm{F}] = \begin{bmatrix} F_{xx} & F_{xy} & F_{xz} \\ F_{yx} & F_{yy} & F_{yz} \\ F_{zx} & F_{zy} & F_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial X} & \frac{\partial x}{\partial Y} & \frac{\partial x}{\partial Z} \\ \frac{\partial y}{\partial X} & \frac{\partial y}{\partial Y} & \frac{\partial y}{\partial Z} \\ \frac{\partial z}{\partial X} & \frac{\partial z}{\partial Y} & \frac{\partial z}{\partial Z} \end{bmatrix} \quad (5.17)$$

Ratios of initial and final material line length components
Stretch, rotation, and shear of infinitesimal material lines

*For all we assume locally homogeneous*

## Small Strain Tensor

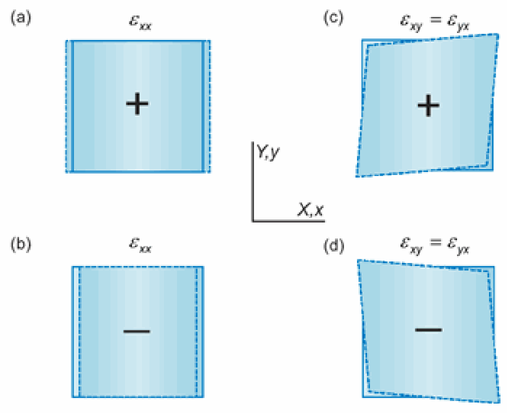



Figure 4.34 Sign conventions for small strain components. Square with solid border represents the initial shape; dashed border represents final state. (a) Stretching is positive; (b) shortening is negative; (c) decrease in a right angle is positive; (d) increase in a right angle is negative.

dimensions are found by the same procedures used above, so no new concepts are required. The small normal strains are:

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial X}, \varepsilon_{yy} = \frac{\partial u_y}{\partial Y}, \varepsilon_{zz} = \frac{\partial u_z}{\partial Z} \quad (4.44)$$

The small shear strains are:

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial Y} + \frac{\partial u_y}{\partial X}\right), \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial Z} + \frac{\partial u_z}{\partial Y}\right), \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial u_z}{\partial X} + \frac{\partial u_x}{\partial Z}\right)$$
$$\varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial u_y}{\partial X} + \frac{\partial u_x}{\partial Y}\right), \varepsilon_{zy} = \frac{1}{2}\left(\frac{\partial u_z}{\partial Y} + \frac{\partial u_y}{\partial Z}\right), \varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial Z} + \frac{\partial u_z}{\partial X}\right) \quad (4.45)$$

Taken together (4.44) and (4.45) are referred to as kinematic equations for small strains.

The small strains are components of the tensor ε, and they may be organized into a square matrix with three rows and three columns:

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \quad (4.46)$$

Displacement derivatives:
No rotational deformation and adequate for strains <$10^{-2}$ to $10^{-3}$

## Finite Strain Tensor

$$[\mathrm{E}] = \begin{bmatrix} E_{xx} & E_{xy} & E_{xz} \\ E_{yx} & E_{yy} & E_{yz} \\ E_{zx} & E_{zy} & E_{zz} \end{bmatrix} \quad (5.18)$$

The primary diagonal elements of the finite strain tensor are related to the spatial derivatives of the displacement components as:

$$E_{xx} = \frac{\partial u_x}{\partial X} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial X}\right)^2 + \left(\frac{\partial u_y}{\partial X}\right)^2 + \left(\frac{\partial u_z}{\partial X}\right)^2\right]$$
$$E_{yy} = \frac{\partial u_y}{\partial Y} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial Y}\right)^2 + \left(\frac{\partial u_y}{\partial Y}\right)^2 + \left(\frac{\partial u_z}{\partial Y}\right)^2\right] \quad (5.19)$$
$$E_{zz} = \frac{\partial u_z}{\partial Z} + \frac{1}{2}\left[\left(\frac{\partial u_x}{\partial Z}\right)^2 + \left(\frac{\partial u_y}{\partial Z}\right)^2 + \left(\frac{\partial u_z}{\partial Z}\right)^2\right]$$

The finite strain tensor is symmetric, so the respective secondary diagonal elements are equal, and they are related to the spatial derivatives of the displacement components as:

$$E_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial Y} + \frac{\partial u_y}{\partial X}\right) + \frac{1}{2}\left(\frac{\partial u_x}{\partial X}\frac{\partial u_x}{\partial Y} + \frac{\partial u_y}{\partial X}\frac{\partial u_y}{\partial Y} + \frac{\partial u_z}{\partial X}\frac{\partial u_z}{\partial Y}\right) = E_{yx}$$
$$E_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial Z} + \frac{\partial u_z}{\partial Y}\right) + \frac{1}{2}\left(\frac{\partial u_x}{\partial Y}\frac{\partial u_x}{\partial Z} + \frac{\partial u_y}{\partial Y}\frac{\partial u_y}{\partial Z} + \frac{\partial u_z}{\partial Y}\frac{\partial u_z}{\partial Z}\right) = E_{zy}$$
$$E_{zx} = \frac{1}{2}\left(\frac{\partial u_z}{\partial X} + \frac{\partial u_x}{\partial Z}\right) + \frac{1}{2}\left(\frac{\partial u_x}{\partial Z}\frac{\partial u_x}{\partial X} + \frac{\partial u_y}{\partial Z}\frac{\partial u_y}{\partial X} + \frac{\partial u_z}{\partial Z}\frac{\partial u_z}{\partial X}\right) = E_{xz} \quad (5.20)$$

Displacement derivatives:
No rotational deformation with no approximation