Advanced Structural Geology, Fall 2022

Simple Rheology

Ramón Arrowsmith

ramon.arrowsmith@asu.edu

ASU School of Earth and Space Exploration
Arizona State University

-What are the three styles of deformation?

- Brittle--Discrete failure accommodates deformation; formation of faults and fractures
- Ductile—Distortion without fracture; distributed deformation
- Viscous—Flow (water, magma)

Rheology:
"science of deformation and flow of matter"
or
relationship between stress and strain

Elastic
Plastic
Viscous

Idealized Elastic Material

- Linear relationship between force and extension (Hooke, 1676):
  - C e l l l n o s s s t t u u
  - Ut tensio sic uis
  - As extension so the force

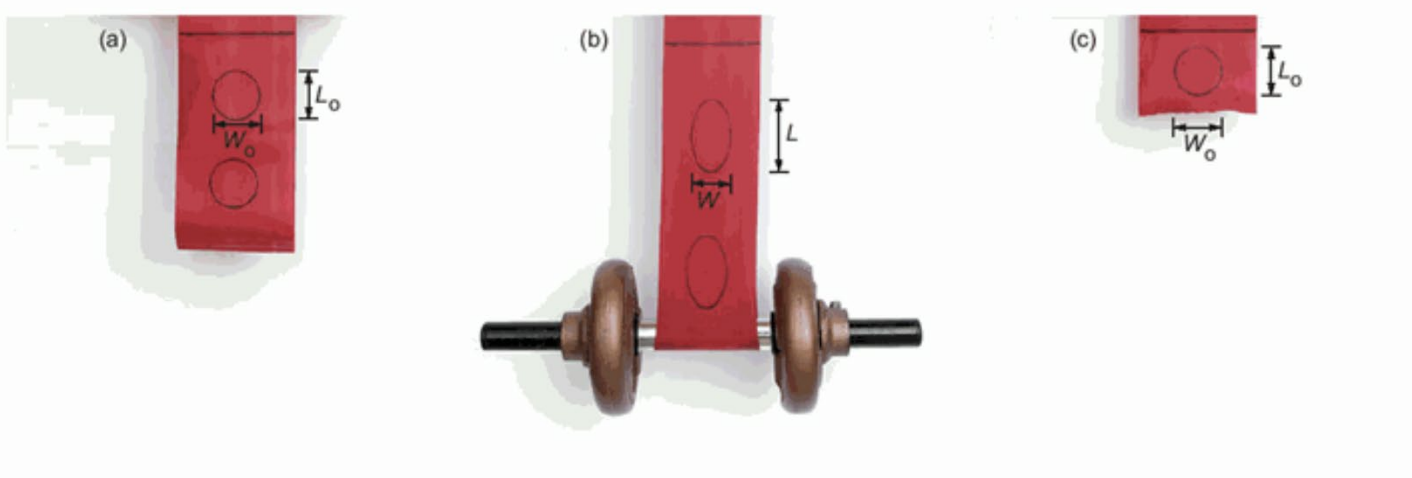
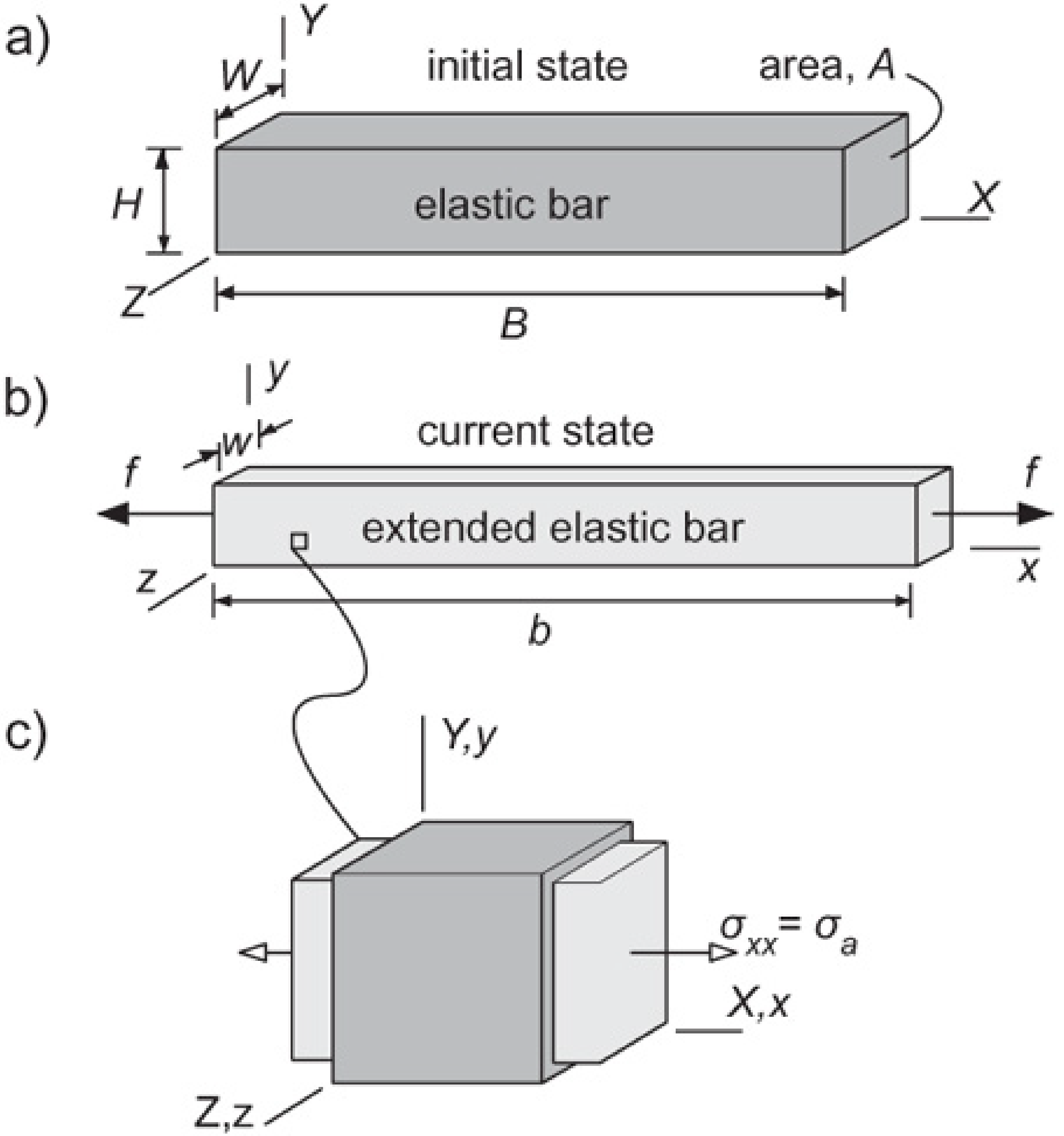


Figure 4.1 Three stages in the brittle elastic deformation of a rubber band. (a) Unloaded band with circles and reference line. (b) Band loaded with a weight stretches such that length increases and width decreases: circles become ellipses. (c) Upper half of fractured band. Both halves return to unloaded configuration and retain elasticity. Photography by Richard Stultz.

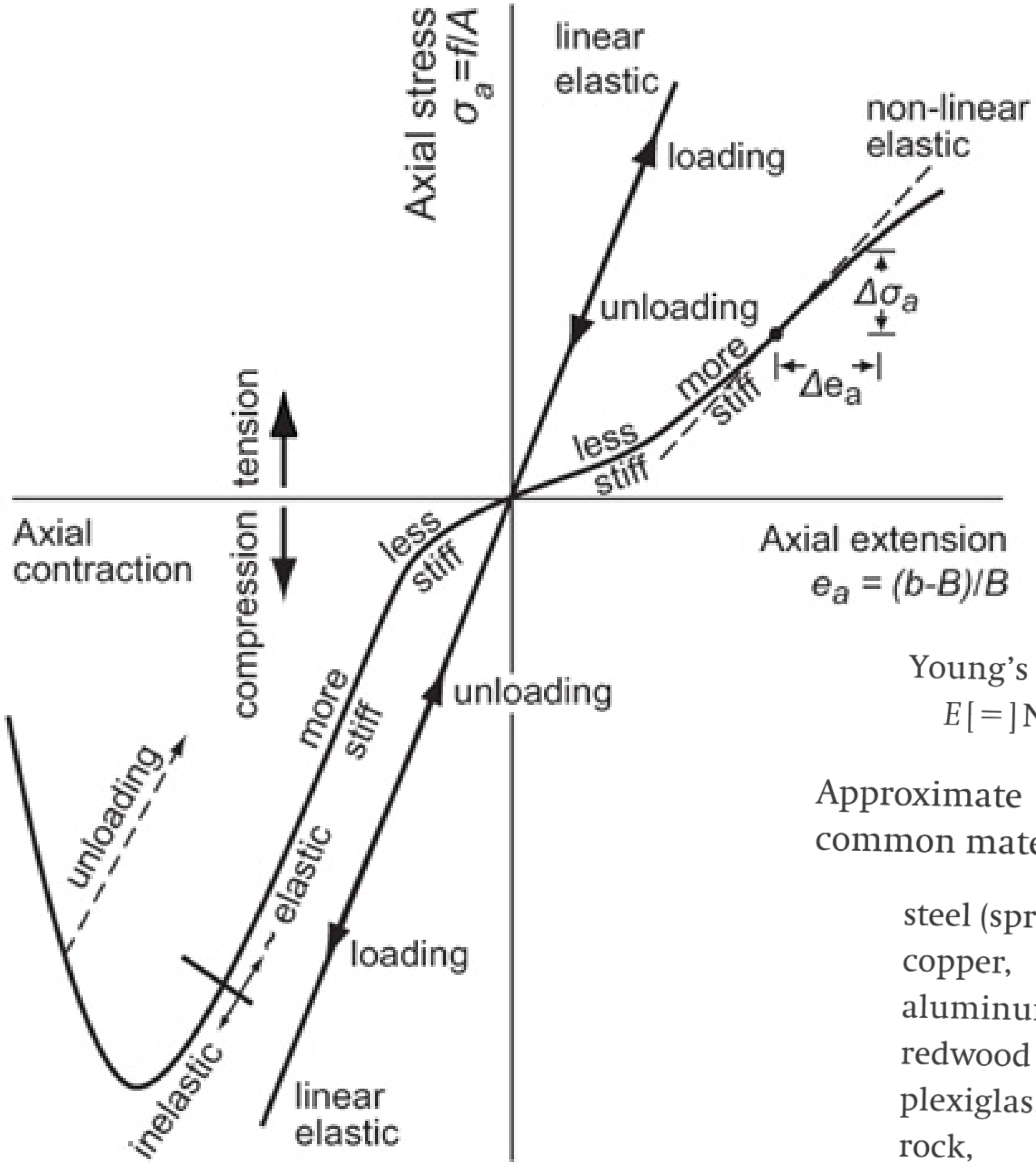


$\sigma_a = f/A$
$e_a = (b-B)/B$

$\sigma_p = 0$
$e_p = (w-W)/W$

$\nu = - e_p / e_a$
(Poisson ratio)
Measures compressibility

rubber,	$\nu \sim 0.5$
cork,	$\nu \sim 0.0$
rock,	$\nu \sim 0.1$ to 0.3



E (young's modulus) = d σ_a/d
e_a
Units of stress

Young's modulus, $E\{=\}ML^{-1}T^{-2}$, and
$E[=]Nm^{-2} = Pa$ (8.8)

Approximate values of Young's modulus for common materials are (Eshbach, 1961):

steel (spring),	$E \sim 200$ GPa
copper,	$E \sim 110$ GPa
aluminum,	$E \sim 70$ GPa
redwood (dry),	$E \sim 9$ GPa
plexiglas,	$E \sim 3$ GPa
rock,	$E \sim 1$ to 100 GPa

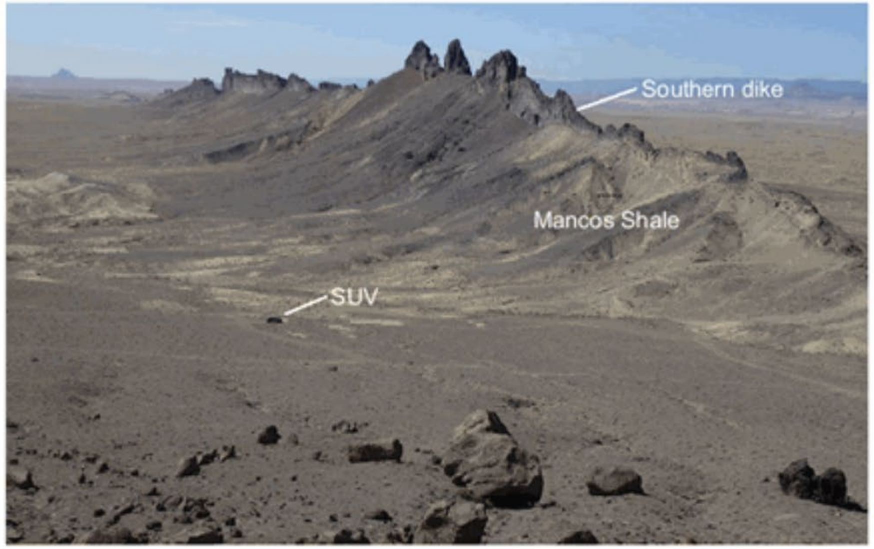


Figure 4.10 Southern dike at Ship Rock, NM is roughly 9 km long and attains a thickness of about 10 m. For scale note the sport utility vehicle (SUV) on the dirt road to left and just below the center of this image. UTM: 12 S 693217.94 m E, 4062057.75 m N.

The maximum opening is proportional to the crack length and to the difference between the pressure and stress. The opening is inversely proportional to the elastic stiffness. Because Poisson's ratio is about ¼, we take $(1-\nu^2) \approx 1$. Using this approximation, and solving (4.9) for Young's modulus we have:

$$E \approx \frac{2(P-S)}{T_m/L} \tag{4.10}$$

Both of the geometric quantities in (4.10) can be measured in the field, and for the south dike at Ship Rock we have $L \approx 9$ km and $T_m \approx 10$ m. Young's modulus in (4.10) is proportional to the difference between the pressure, P, and the compressive stress, S. To estimate these quantities we assume the dike breached Earth's surface 1 km above the outcrop. The magma pressure is estimated as that in a static column of magma 1 km tall: $P = \gamma_m D$ where $\gamma_m \approx 2.6 \times 10^4$ N m^{-3} is the unit weight of the magma and $D = 1$ km. Similarly, the least compressive stress is estimated as $S = \gamma_r D$ where $\gamma_r \approx 2.4 \times 10^4$ N m^{-3} is the unit weight of Mancos Shale. Using these estimates, the driving pressure for the south dike is $P - S \approx (\gamma_m - \gamma_r)D \approx 2$ MPa. Using these values in (4.10), the estimated Young's modulus of the sedimentary rock at Ship Rock is $E \approx 4$ GPa.

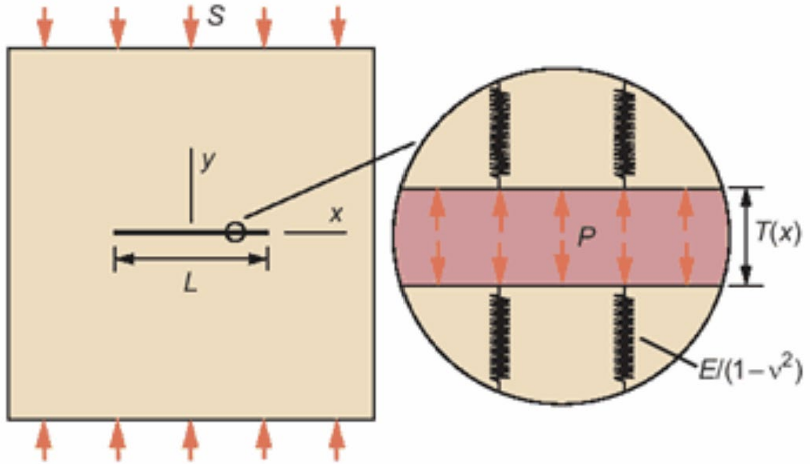


Figure 4.11 Conceptual model of a crack of length, L, and opening $T(x)$ in an elastic material. Fluid pressure, P, acts inside the crack (see inset), while a remotely applied compressive stress of magnitude S acts across the plane of the crack. Elastic stiffness is $E/(1-\nu^2)$, represented schematically by springs.

Youngs Modulus estimate at this scale is relatively low

But that is because the rock is not uniform but has flaws, voids, other fractures, etc. that make it softer.

Idealized Elastic Material

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right]$$

Stiffness and strength

“Lest there be any possible, probably, shadow of doubt, strength is not, repeat not, the same thing as stiffness. Stiffness, or Young’s modulus or E, is concerned with how stiff, flexible, springy or floppy a material is. Strenght is the force or stress needed to break a thing. A biscuit is stiff but weak, steel is stiff and strong, nylon is flexible (low E and strong), raspberry jelly is flexible (low E) and weak. The two properties together describe a solid about as well as you can reasonably expect two figures to do. (Gordon)

Plasticity

- Steady deformation at yield strength

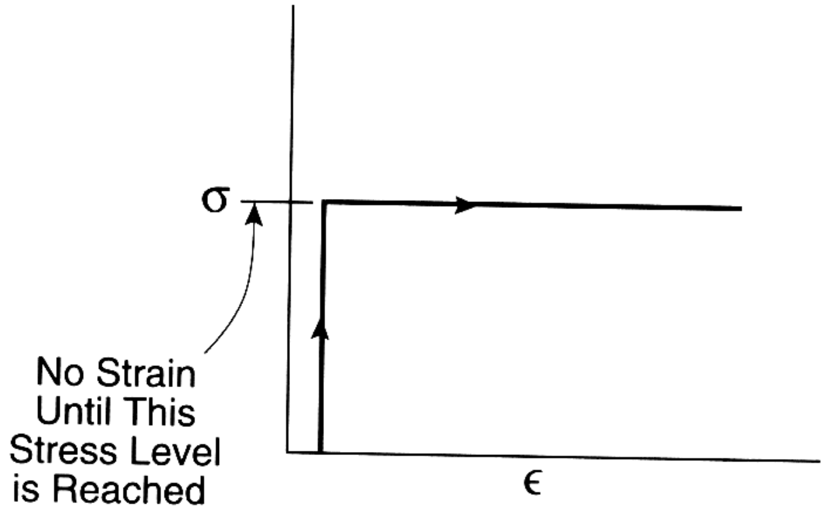


Figure 3.53 Portrayal of ideally plastic deformation. Stress (σ) is raised, but no strain (ϵ) accrues until a critical threshold is exceeded. From that point on, under ideal conditions, deformation continues as long as the stress level is maintained.

-Davis and Reynolds

Viscosity

- Strain rate proportional to differential stress

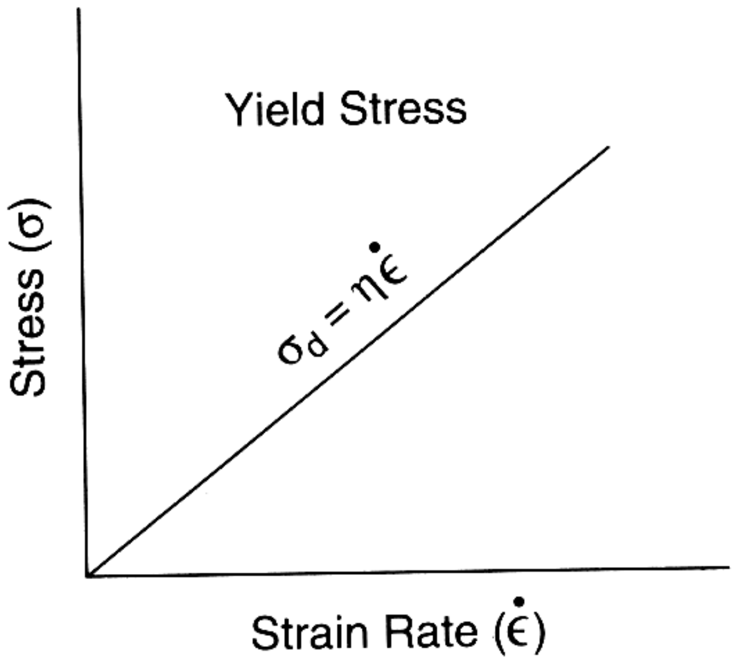


Figure 3.55 Portrayal of ideal viscous behavior on plot of stress (σ) versus strain rate (ϵ).

-Davis and Reynolds

where σ_d = differential stress

η = viscosity

$\dot{\epsilon}$ = strain rate