Advanced Structural Geology, Fall 2022

Force, Tractions, Stress

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Some from Davis, Reynolds, Kluth, Structural Geology of Rocks and Regions

School of Earth and Space Exploration

Arizona State University

Stress simply

Simple definition of stress (for the moment) = pressure =Force/Area

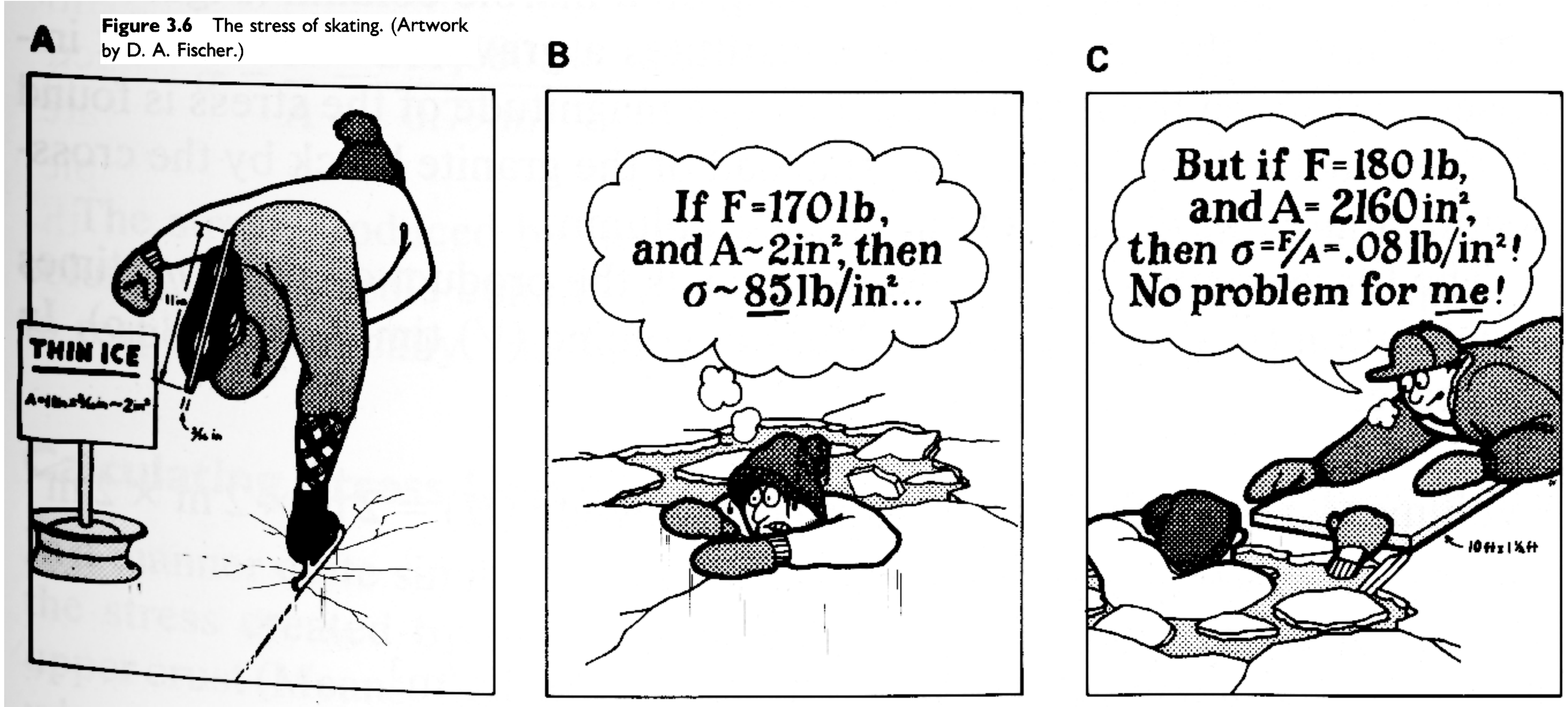


Figure 3.6 The stress of skating. (Artwork by D. A. Fischer.)

Stress simply

Stress = Force/Area = N/m^2 = Pascal = Pa

Blaise Pascal, was a French mathematician, physicist, inventor, writer and Christian philosopher. Pascal's earliest work was in the natural and applied sciences where he made important contributions to the study of fluids, and clarified the concepts of pressure and vacuum. Pascal also wrote in defense of the scientific method.

Born: June 19, 1623, Clermont-Ferrand

Died: August 19, 1662, Paris

(Wikipedia)

Pascal aside

In his Lettres Provinciales, the French philosopher and mathematician Blaise Pascal famously wrote:

I would have written a shorter letter, but I did not have the time.

This sentiment, which also found expression by John Locke, Benjamin Franklin and Woodrow Wilson, among others, reflects both the value and the challenges of brevity. On the one hand, brevity forces us to cut excess — to distill a message to its core. On the other hand, being accurate, clear and concise can be hard, sometimes impossibly so.

https://www.npr.org/sections/13.7/2014/02/03/270680304/this-could-have-been-shorter

Stress and deformation

- Use your stress and deformation words correctly:

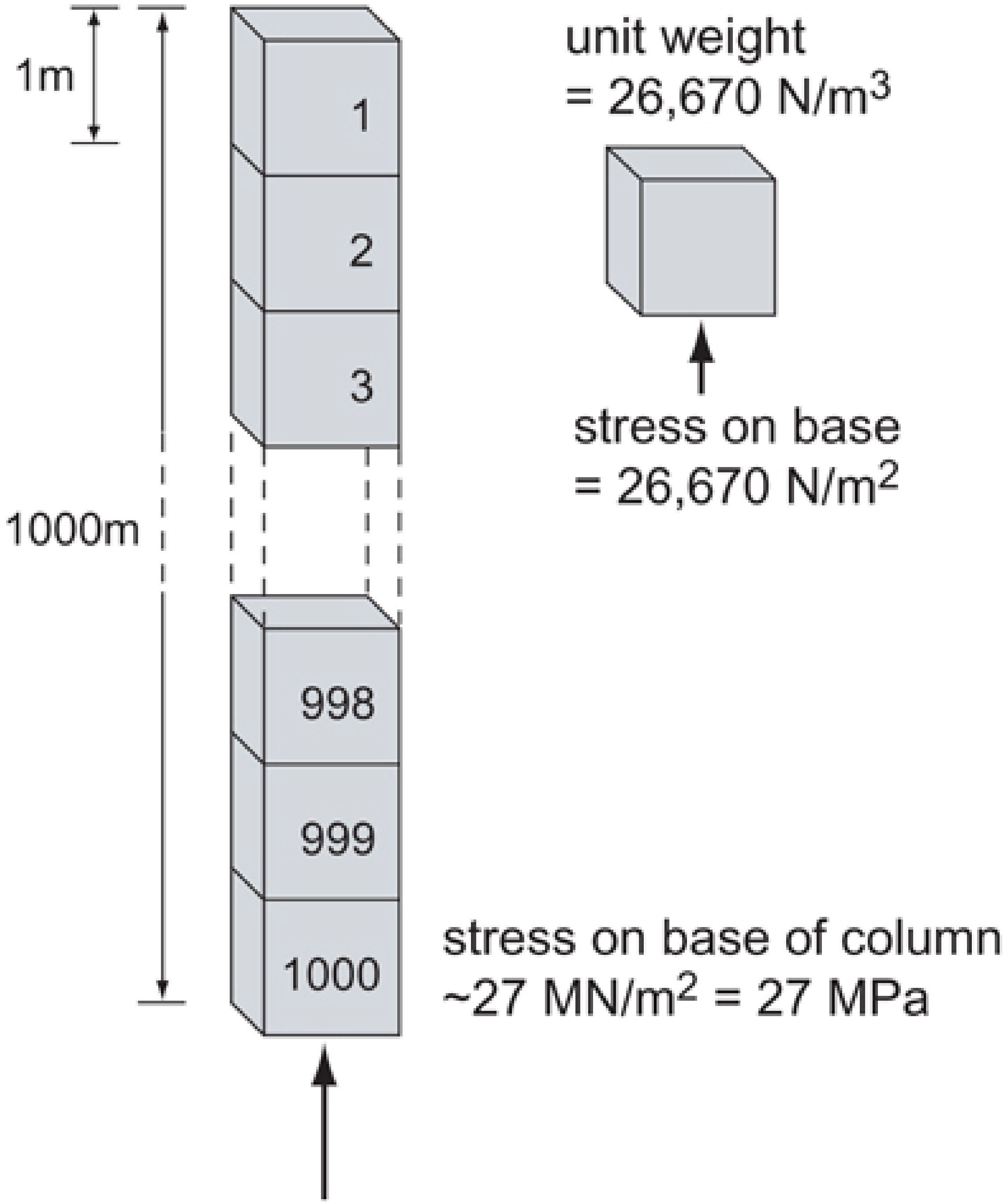
• Stress	Deformation (strain)
• Tension	Extension
• Compression	Contraction or shortening

• Many writers of geologic literature use "compression" indiscriminately for both stress and strain, as in the context of "compressional structures." Geologic structures are manifestations of *strain*; thus, in rock mechanics the convention is that "tension" and "compression" are terms that should be used in discussions of *stress*, whereas the corresponding *strain* terms are "extention" or "elongation," and "contraction" or "shortening" or even "constriction."

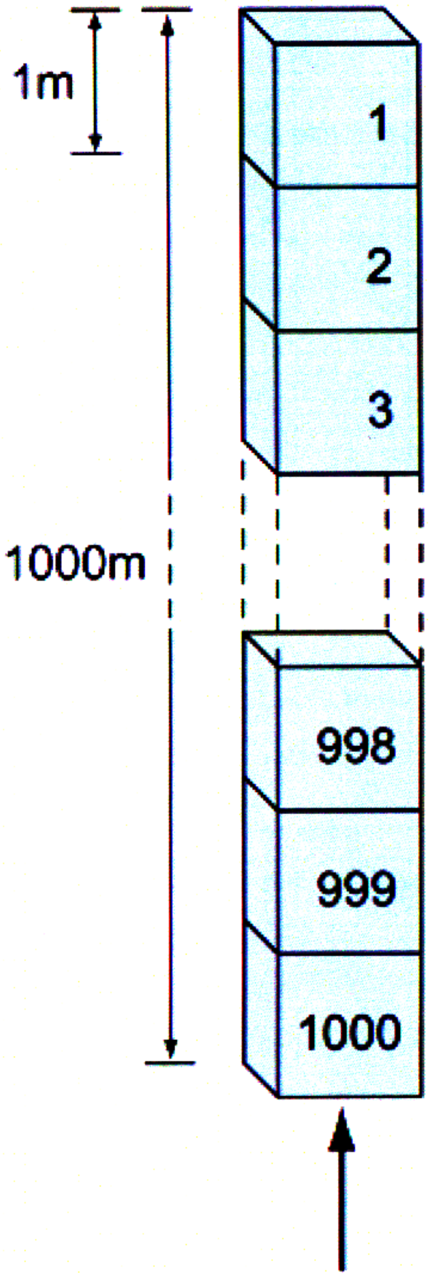
-watch out for Art Sylvester!

http://www.geol.ucsb.edu/faculty/sylvester/

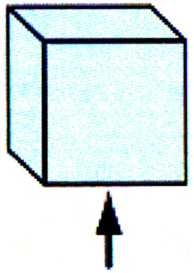
Ray Mine AZ: rocks come from depth "overburden"


1m
1000m
1
2
3
998
999
1000
unit weight
= 26,670 N/m3
stress on base
= 26,670 N/m2
stress on base of column
~27 MN/m2 = 27 MPa

V = volume



unit weight
$= 26{,}670\ N/m^3$



stress on base
$= 26{,}670\ N/m^2$

stress on base of column
$\sim 27\ MN/m^2 = 27\ MPa$

$\vec{F} = m\vec{a}$; $\vec{F} = m\vec{g}$

$\vec{g} = 9.81\ m/s^2 \approx 10\ m/s^2$

stress at base $= \frac{\vec{F}}{A}$

rock density $= 2667\ kg/m^3$

$\rho = \frac{m}{V} \Rightarrow m = \rho \cdot V$

$m = 2667\ kg/m^3 \cdot m^3 = 2667\ kg$

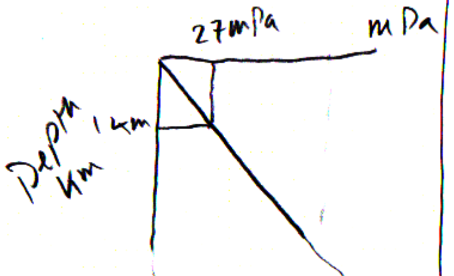
$\vec{F} = m\vec{g} = 2667\ kg \cdot 10\ m/s^2 = 2.667 \times 10^4\ N$

Area $= 1\ m^2$

$\vec{F}/A = 2.667 \times 10^4\ N/m^2$
$= 2.667 \times 10^4\ Pa$

1km tall column of rock:
$2.667 \times 10^4\ Pa \times 1000 = 2.667 \times 10^7\ Pa$
$= 26.67\ MPa/km$

vertical stress gradient

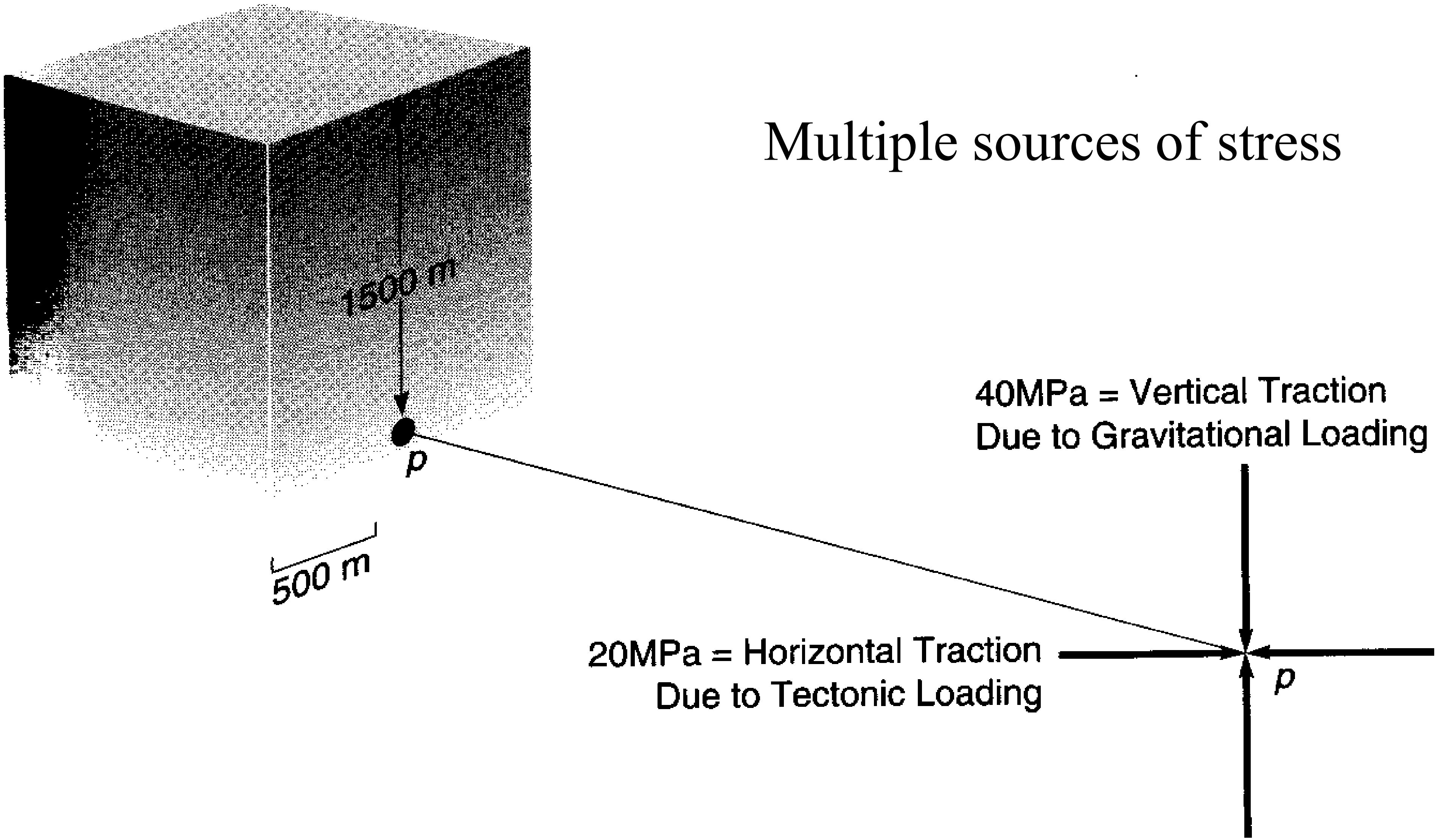


Diamond: 100km deep

$27\ MPa \cdot 100\ km$

2.7 GPa

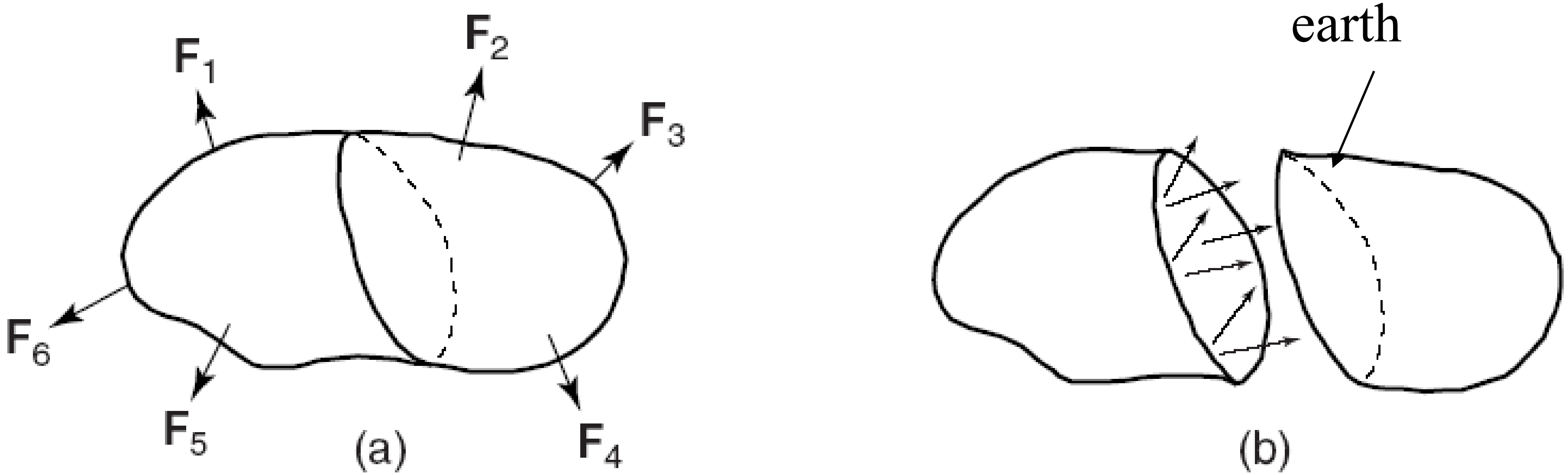
Multiple sources of stress



Tractions and stress

Replace effect of adjacent rocks with forces

Rock quarried from the earth



Applied forces: (a) static equilibrium; (b) forces acting on a plane.

Tractions and stress

A more strict definition of stress

- Traction is stress relative to a surface through a point p.
- Stress tensor is the field of tractions acting over a point p.
- Stress field is the entire collection of stress tensors in a body.

Tractions and stress

- Start with equilibrium (Newton's 3rd Law):

- Resolve each into its components in the coordinate directions

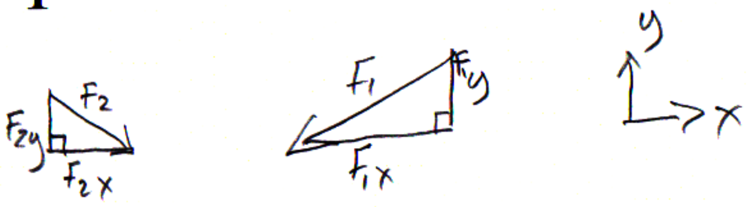
Tractions and stress

- Start with equilibrium (Newton's 3rd Law):

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = 0$$

- Resolve each into its components in the coordinate directions

$$\sum \vec{F}_x = 0 \quad \sum \vec{F}_y = 0 \quad \sum \vec{F}_z = 0$$



Tractions and stress

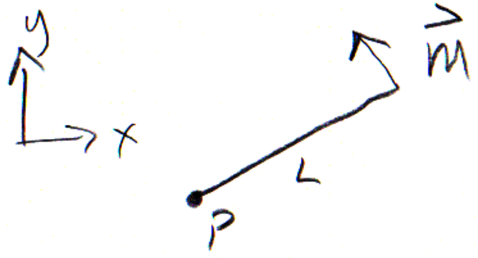
- Total torque must vanish

Tractions and stress

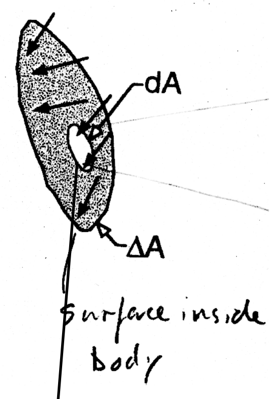
- Total torque must vanish

$$\vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \cdots = 0$$

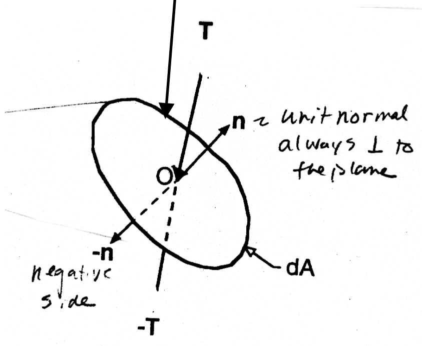
$$\Sigma \vec{M}_x = 0 \quad \Sigma \vec{M}_y = 0 \quad \Sigma \vec{M}_z = 0$$



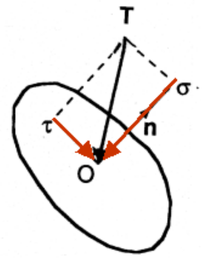
Tractions and stress



(a) "Traction" or "Stress vector"



(b) Equilibrium "Cauchy's relationship



(c) Components of the Traction Vector

Tractions and stress

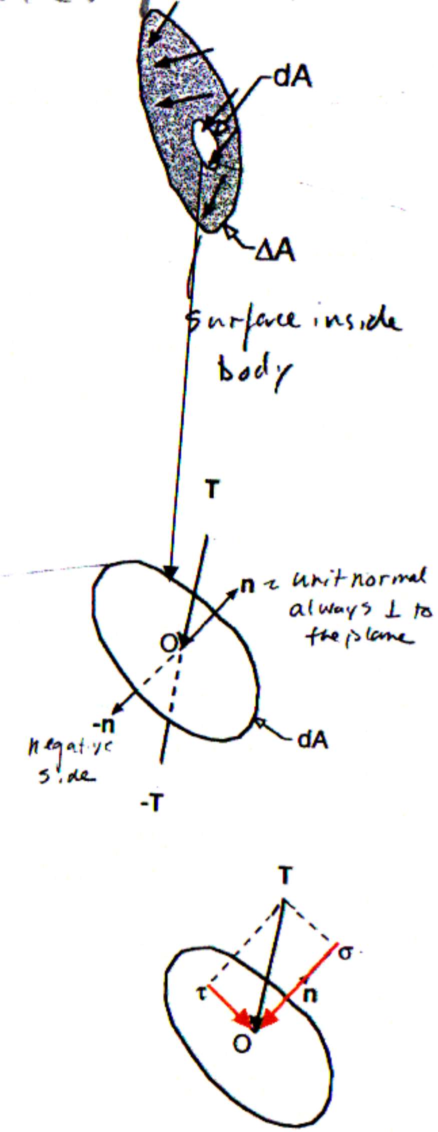
traction as a function of orientation of surface defined by normal vector

$$\vec{T}(\vec{n}) = \lim_{\Delta A \to 0} \frac{\Delta \vec{F}}{\Delta A} = \frac{d\vec{F}}{dA}$$

$$\vec{T}(\vec{n}) + \vec{T}(-\vec{n}) = 0$$

σ_n or σ is normal component

σ_s or τ is shear component



(a) "Traction" or "Stress vector"

(b) Equilibrium "Cauchy's relationship

(c) Components of the Traction Vector

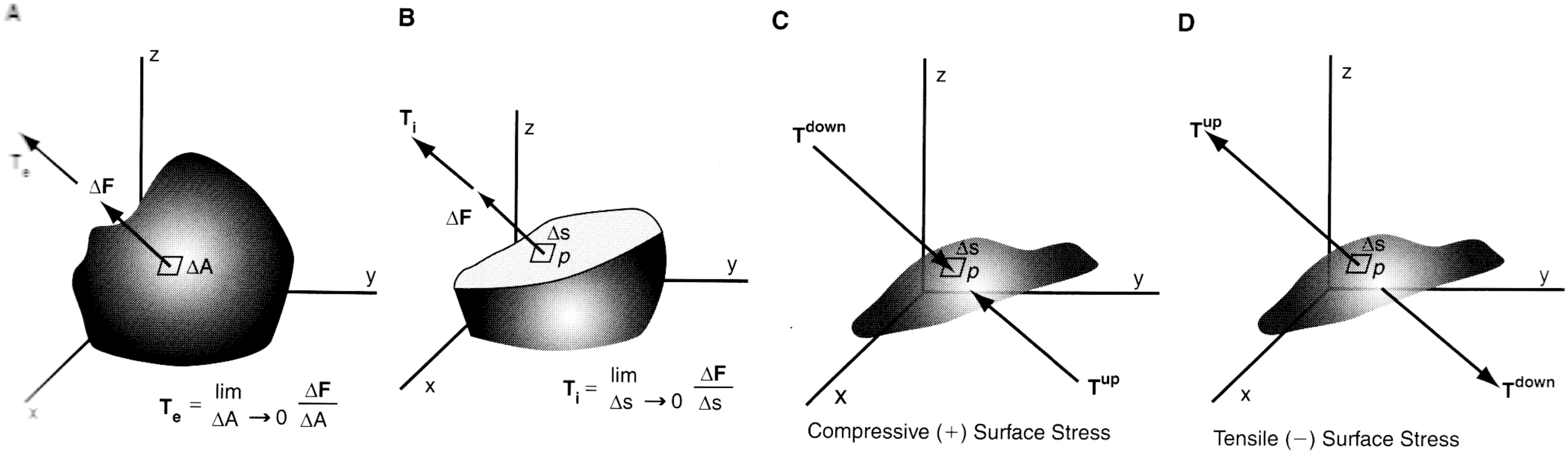


Figure 3.17 (*A*) An external traction (**T_e**) acting on the surface of a body, which we imagine to be a tiny grain of sand in a sandstone formation. Its value is the magnitude of the force (**F**) divided by the area (*A*) on which the force is acting. [Figure prepared courtesy of www.efunda.com, in particular, http://www.efunda.com/formulae/solid_mechanics/mat_mechancis/stress.cfm, April 12, 2010]. (*B*) Internal traction (**T_i**) acting on surface within a body. [Adapted from efunda] [Figure prepared courtesy of www.efunda.com, in particular, www.efunda.com/formulae/solid_mechanics/mat_mechancis/stress.cfm, April 12, 2010]. (*C*) The surface stress at point *p* is the pair of equal and opposite tractions. In this case the surface stress is compressive (+). (*D*) Example of tensile surface stress at point *p*, composed of pair of equal and opposite tractions.

Tractions and stress

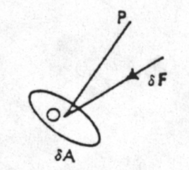
- Sign Conventions: compression positive most of the time

It is now necessary to introduce a convention of sign, and the one which will be used here is that forces are reckoned positive when compressive, that is, in the direction shown by δF in Fig. 2.2 (*a*). This is opposite to the convention adopted in works on the theory of elasticity and continuum

mechanics in which stresses are usually reckoned positive when tensile. In rock mechanics, however, it is more convenient to have compressive stresses positive for the following reasons: (i) the environmental stresses, such as stress due to depth of burial, confining pressure in apparatus, and fluid pressure in pores, are always compressive; (ii) this convention is universal in the closely related subject of soil mechanics, cf. Scott (1963), and has been much used in structural geology; (iii) many problems in rock mechanics involve friction over surfaces, and in this case the normal stress across the surfaces is necessarily compressive. This change of convention leaves all formulae unaltered, but when using results from works on the theory of elasticity (which use the convention that stresses are positive when tensile) it has to be remembered that all signs have to be changed.

-Fundamentals of Rock Mechanics by Jaeger and Cook (3rd ed., p. 10)

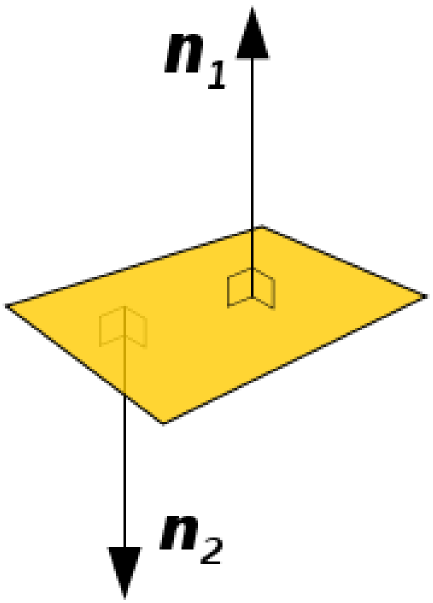
In many engineering situations, we consider tension as positive and compression negative. This is not usually the case in structural geology.



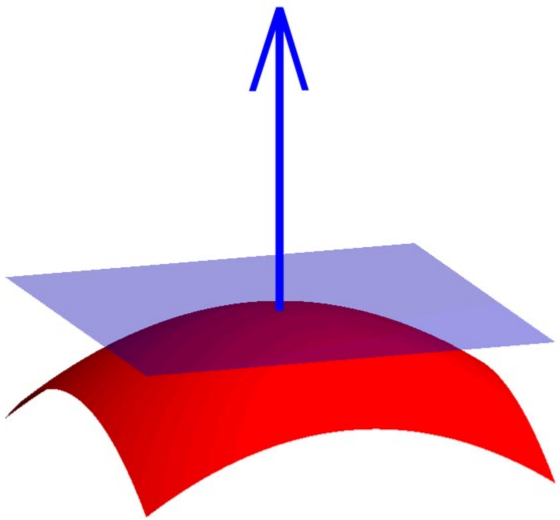
(a)
Traction vector

Fig. 2.2

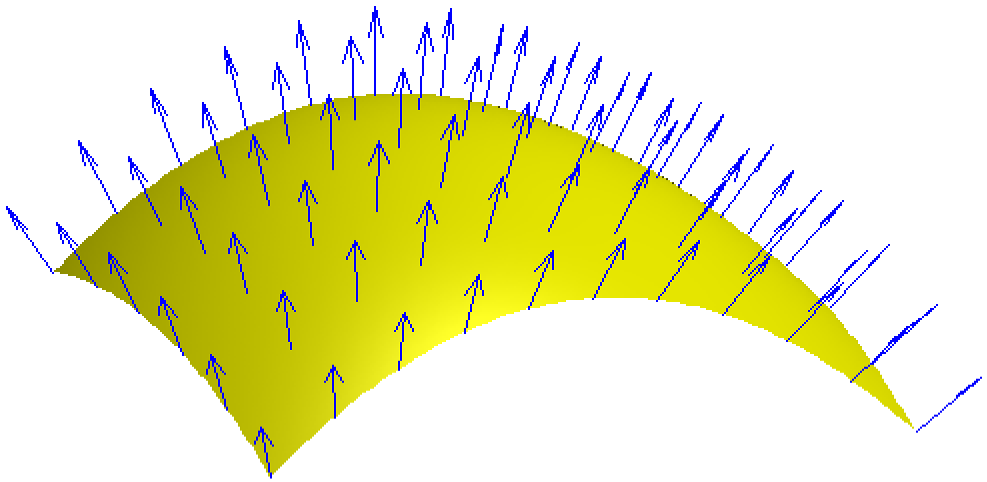
Normal vector: unit length, perpendicular to plane



A polygon and two of its normal vectors



A normal to a surface at a point is the same as a normal to the tangent plane to that surface at that point.

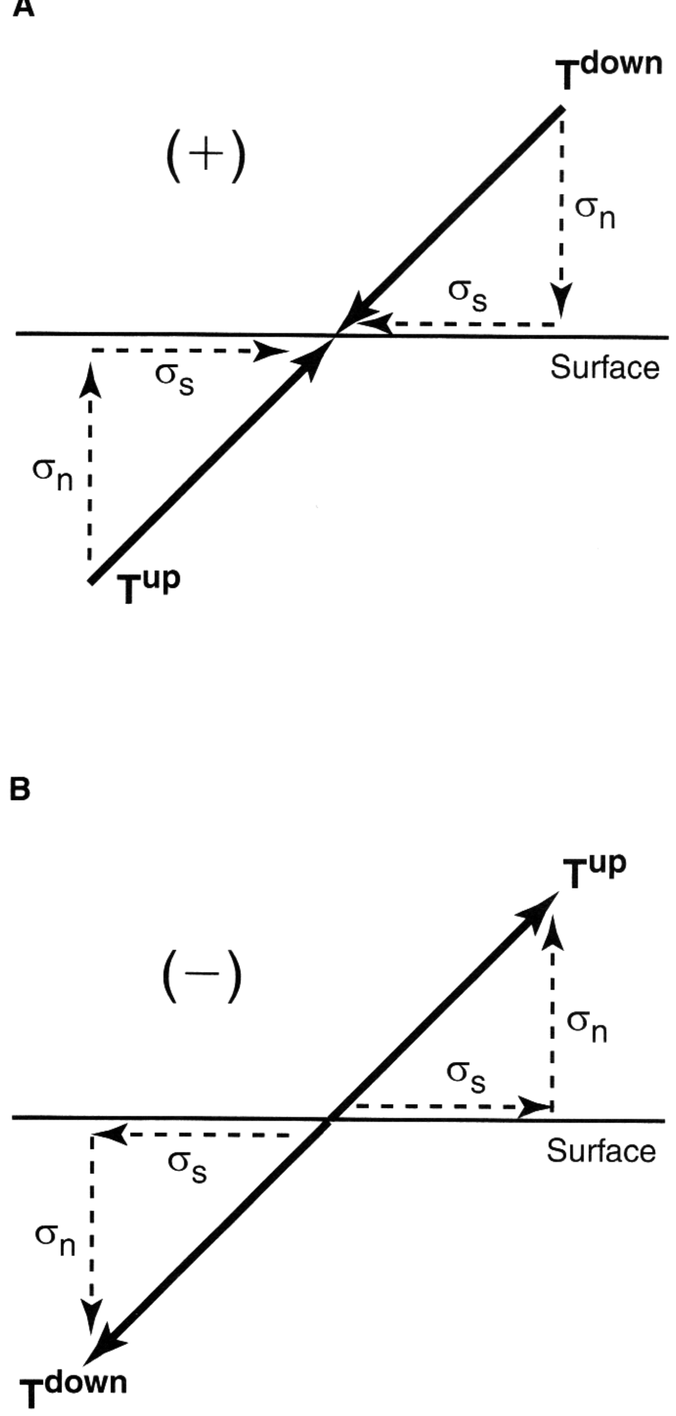


A vector field of normals to a surface

http://en.wikipedia.org/wiki/Normal_(geometry)

Baron Augustin-Louis Cauchy (21 August 1789 – 23 May 1857) was a French mathematician who was an early pioneer of analysis. He started the project of formulating and proving the theorems of infinitesimal calculus in a rigorous manner. A profound mathematician, Cauchy exercised a great influence over his contemporaries and successors. His writings cover the entire range of mathematics and mathematical physics. More concepts and theorems have been named for Cauchy than for any other mathematician (in elasticity alone there are sixteen concepts and theorems named for Cauchy).

Equilibrium



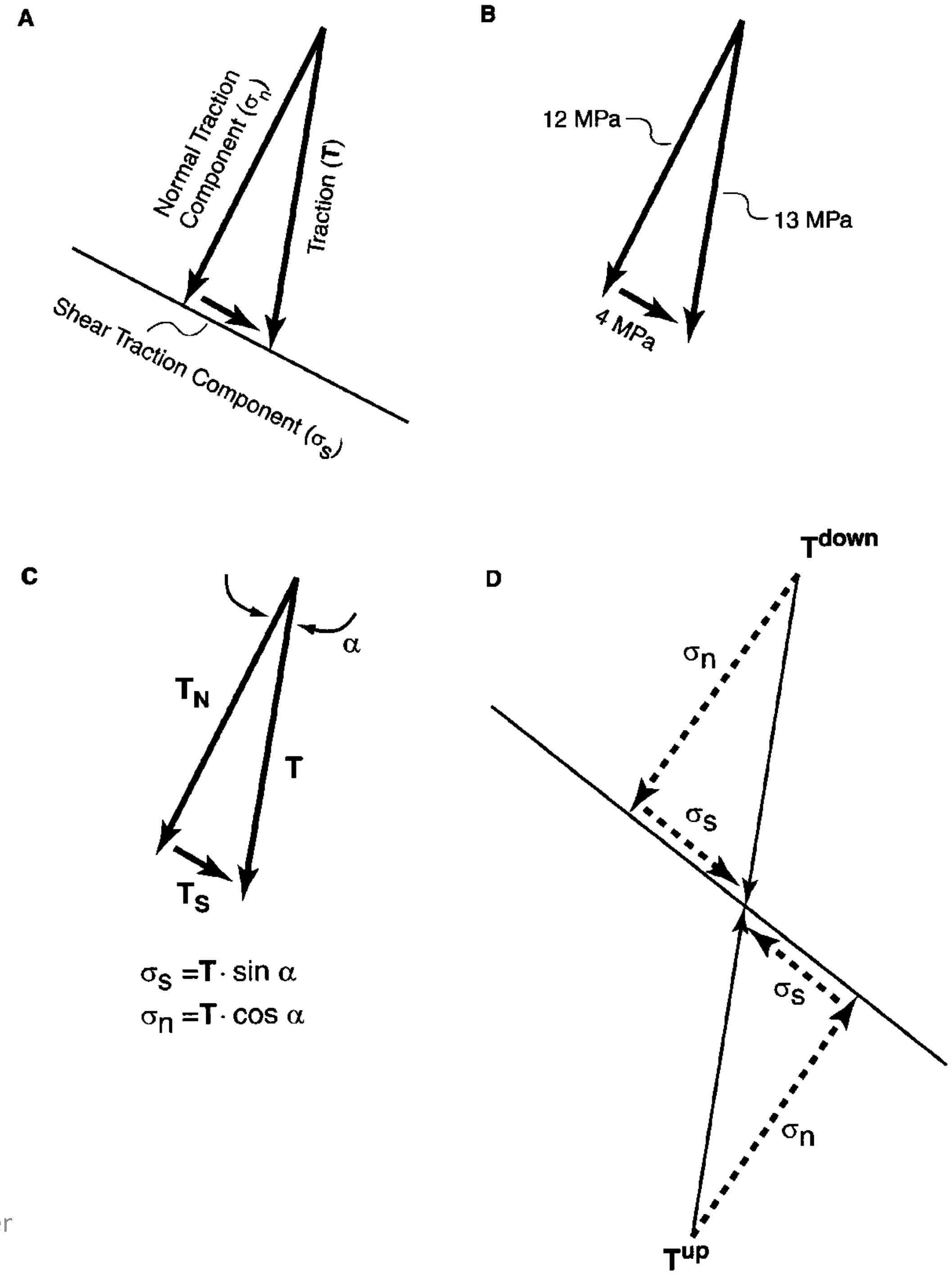


Figure 3.21 Two-dimensional approach to resolving traction (**T**) into normal traction component (σ_n) and shear traction component (σ_s). (*A*) The traction (**T**) in this example is not perpendicular to the surface on which it acts, and thus can be resolved into a normal traction component (σ_n) and shear traction component (σ_s). (*B*) Scaled-drawing solution. (*C*) Trigonometric solution. (*D*) Full surface stress showing resolution of traction.

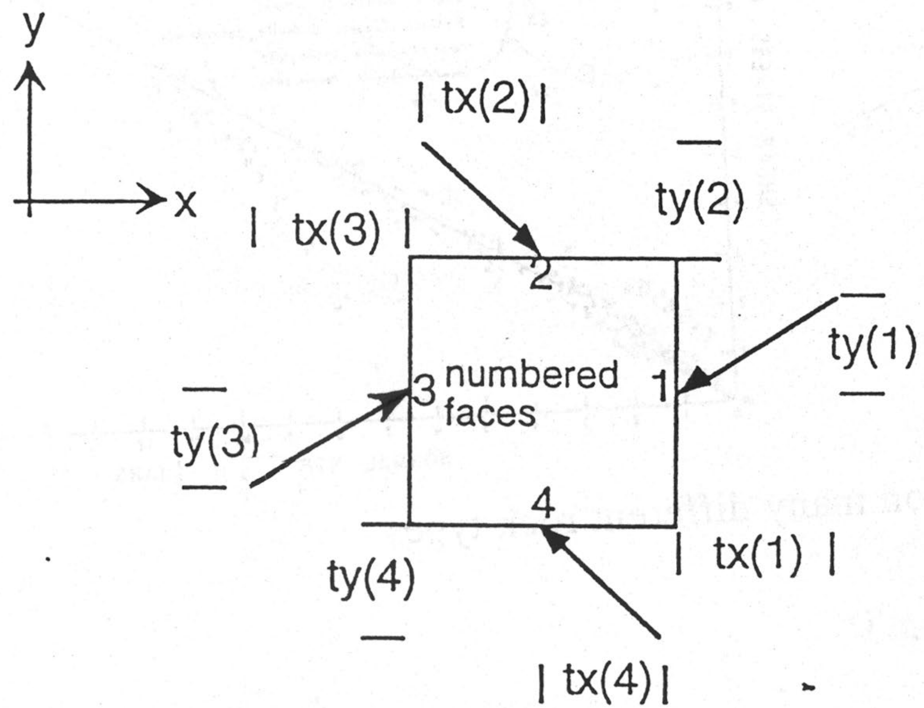
Stress tensor

Start with Cauchy's relationship

$$\vec{t}(\vec{n}) + \vec{t}(-\vec{n}) = 0$$

Consider a cubic element of rock, quarried from the earth with the appropriate boundary tractions replacing the actions of the earth:

Resolve the tractions on the sides into x and y components



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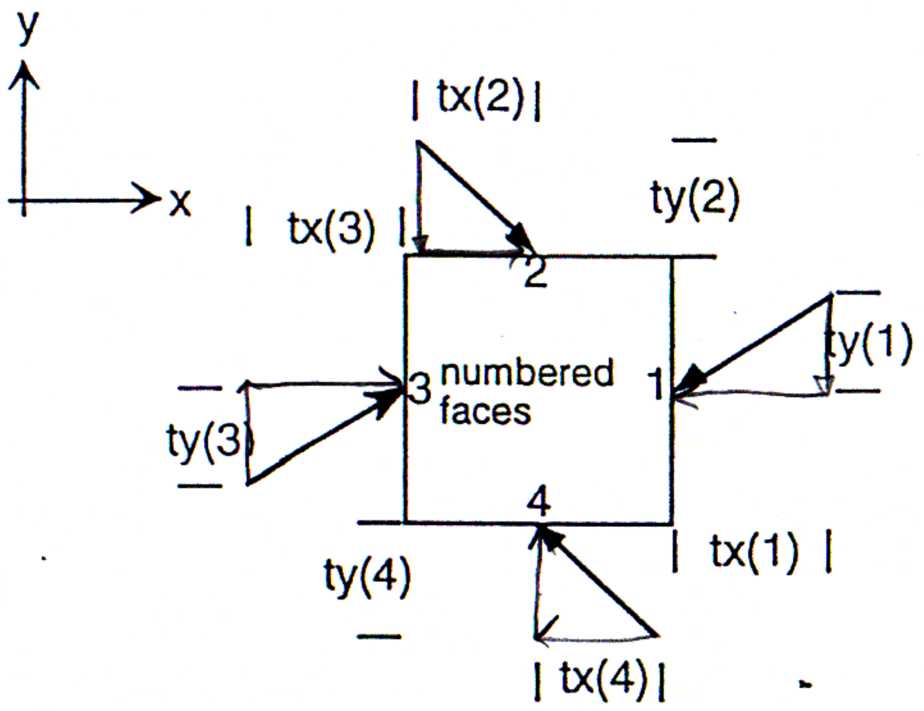
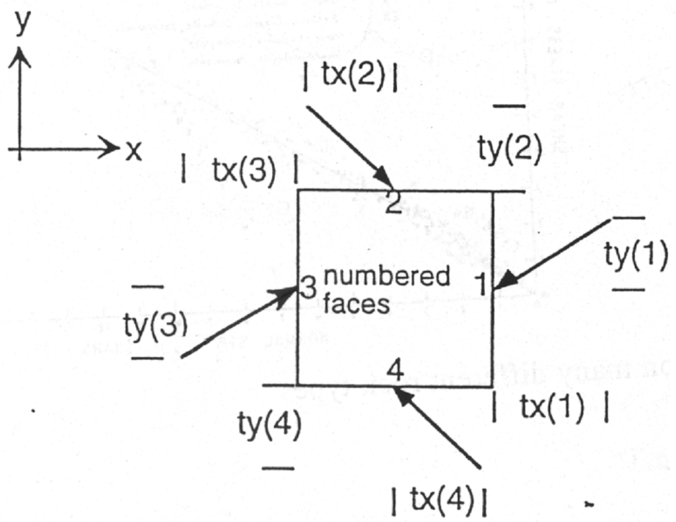


Stress tensor

If the element is small enough, Caucy's relationship will hold and the respective components will be equal in magnitude and opposite in sign.

tx(3) = -tx(1)	Normal component
ty(3) = -ty(1)	Shear component
tx(4) = -tx(2)	Shear component
ty(4) = -ty(2)	Normal component



Stress tensor

If the element is small enough, Caucy's relationship will hold and the respective components will be equal in magnitude and opposite in sign.

tx(3) = -tx(1)	→ σ_{xx}	Normal component
ty(3) = -ty(1)	→ σ_{xy}	Shear component
tx(4) = -tx(2)	→ σ_{yx}	Shear component
ty(4) = -ty(2)	→ σ_{yy}	Normal component

"on - in" convention
on a face
in a coordinate direction

Those fixed relationships let us define the stress tensor components:

[redacted] [redacted]

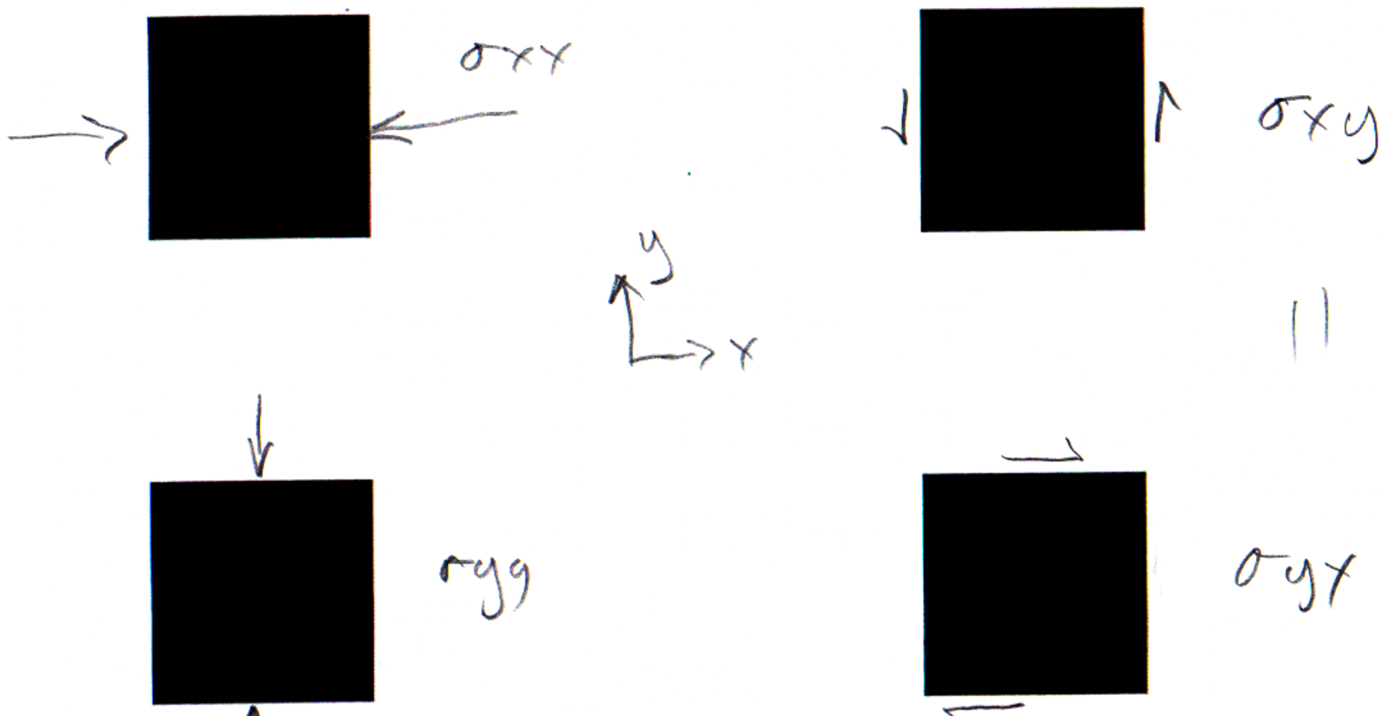
[redacted] [redacted]

$\sigma_{xy} = \sigma_{yx}$ (because torques must sum to zero), so we really only need 3 independent stress components in 2 dimensions:

$$\sigma_{xx}, \sigma_{yy}, \sigma_{xy} = \text{STATE OF STRESS}$$

[redacted]

Those fixed relationships let us define the stress tensor components:



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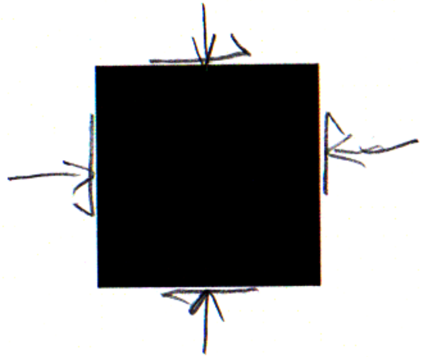
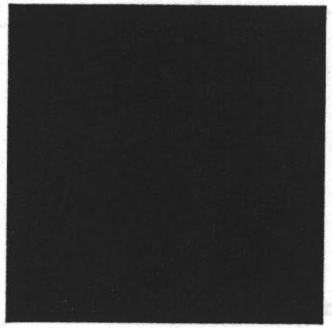


$\sigma_{xy} = \sigma_{yx}$

Stress tensor

Principal stresses

We can always find orientations of a cubic element such that the shear shress components are zero on all sides and in that case, the normal stress components are calle the <u>principal stresses</u>.



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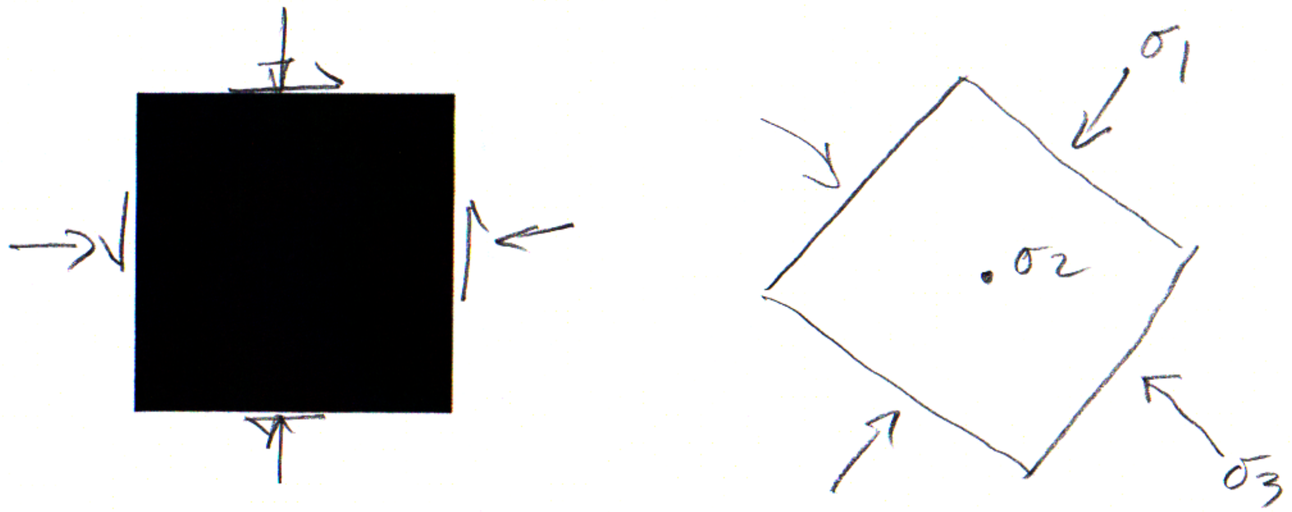

$$\sigma_1 > \sigma_2 > \sigma_3$$

σ_1 = max compression

σ_3 = min compression