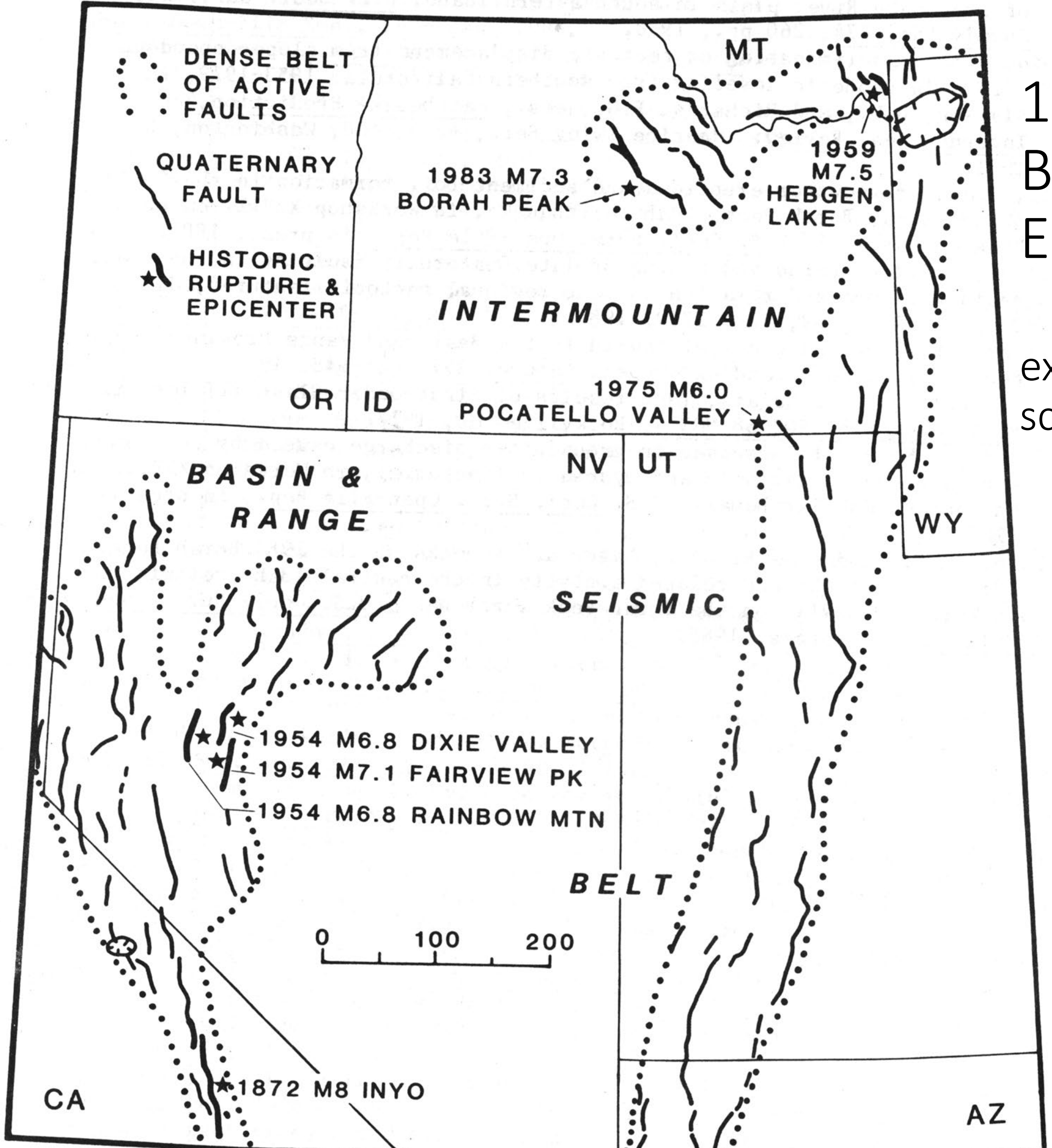
Simple models of fault scarps

Ramón Arrowsmith

ramon.arrowsmith@asu.edu

School of Earth and Space Exploration
Arizona State University

Continuity equation



1983 M7.3 Borah Peak Earthquake

example fault scarps

Example of quarters

1 2 3

Simple mass continuity for a hillslope element:
Material is moving „downhill“ to the right

Consider pile #2

quarters = 8

Rate of input = 2

quarters = 10

Rate of output = 2

quarters = 8

Net change = 0

1 2 3

Consider pile #2

quarters = 8

Rate of input =1

quarters = 9

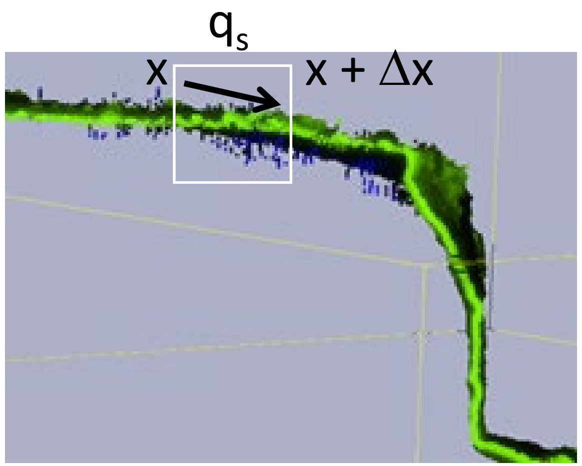
Rate of output = 7

quarters = 2

Net change = -6

1 2 3

For a given very small hillslope element



0 = Rate of removal of volume per unit width downslope - Rate of supply of volume per unit with + Rate of accumulation per unit width

$0 = q_s(x+\Delta x) - q_s(x) + \Delta x \Delta z/\Delta t$

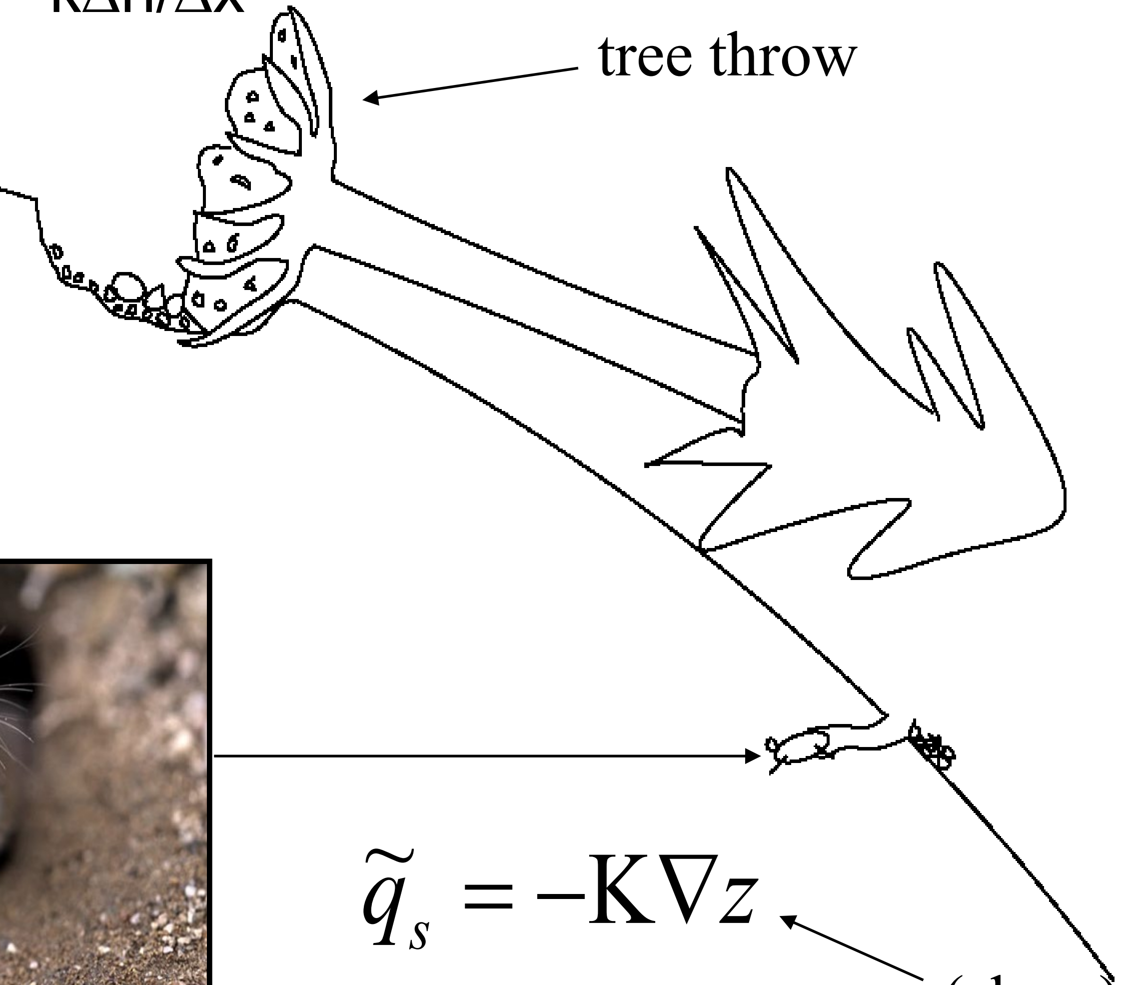
$0 = \Delta q_s + \Delta x \Delta z/\Delta t$ $\quad -\Delta x \Delta z/\Delta t = \Delta q_s$

$\Delta z/\Delta t = -\Delta q_s/\Delta x$

Take the limits $dz/dt = -dq_s/dx$ → “Continuity equation for sediment transport

Biogenic transport—slope dependent

Assume $q_s(x) = -k\Delta h/\Delta x$



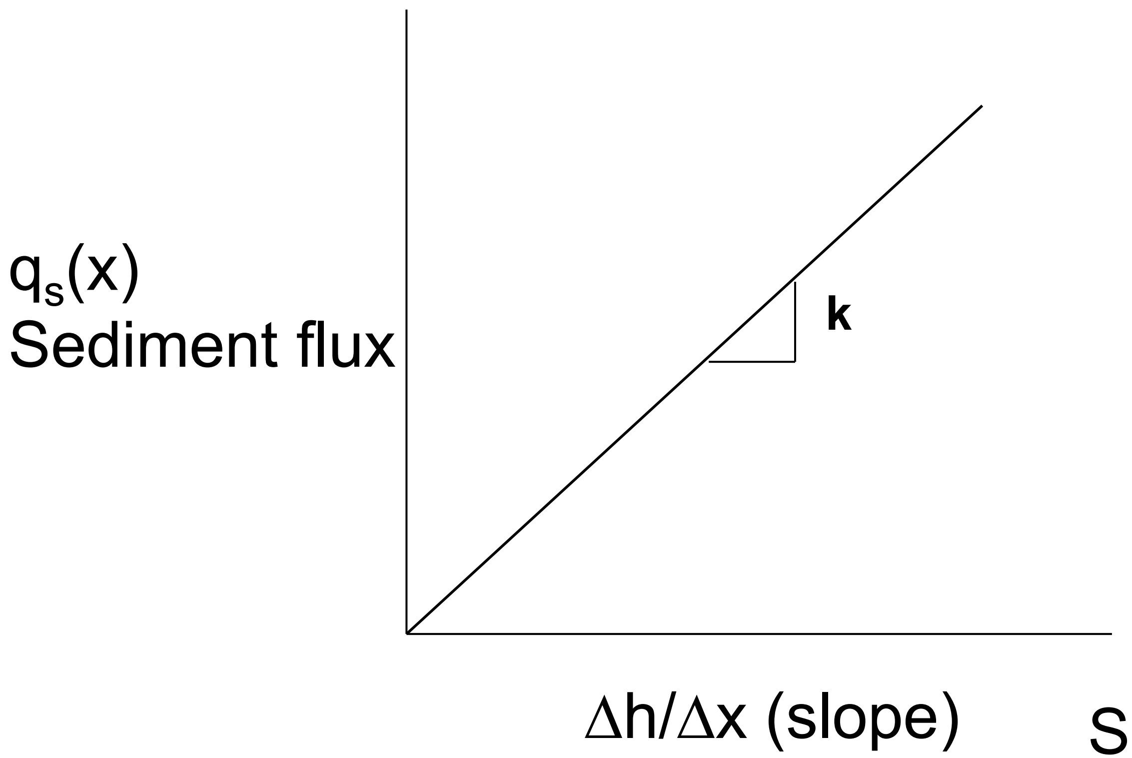
$$\widetilde{q}_s = -K\nabla z$$

burrowing

wdfw.wa.gov/wlm/living/gophers.htm

Modified from DiBiase, 2006

Slope dependent transport law



Soil creep
Biogenic processes (burrowing, other animal induced disturbances)
Rainsplash, etc.

Thus, $q_s(x) = k\Delta h/\Delta x$
Assume k constant in time and space

Combine continuity and transport rule

Comb

Continuity: $\frac{\Delta H}{\Delta T} = -\frac{\Delta q_s}{\Delta x}$

Transport rule: $q_s = -k\frac{\Delta H}{\Delta x}$

Generalized transport rule:

$$q_s = kx^m \nabla h^n$$

Let m = 0 and n=1

So that it is only slope dependent

(no runoff)

Negative sign above for positive transport downslope to the right

$$\frac{\Delta H}{\Delta t} = -\frac{\Delta\left(-k\frac{\Delta H}{\Delta x}\right)}{\Delta x}$$

$$\frac{\Delta H}{\Delta t} = k\frac{\Delta\left(\frac{\Delta H}{\Delta x}\right)}{\Delta x}$$

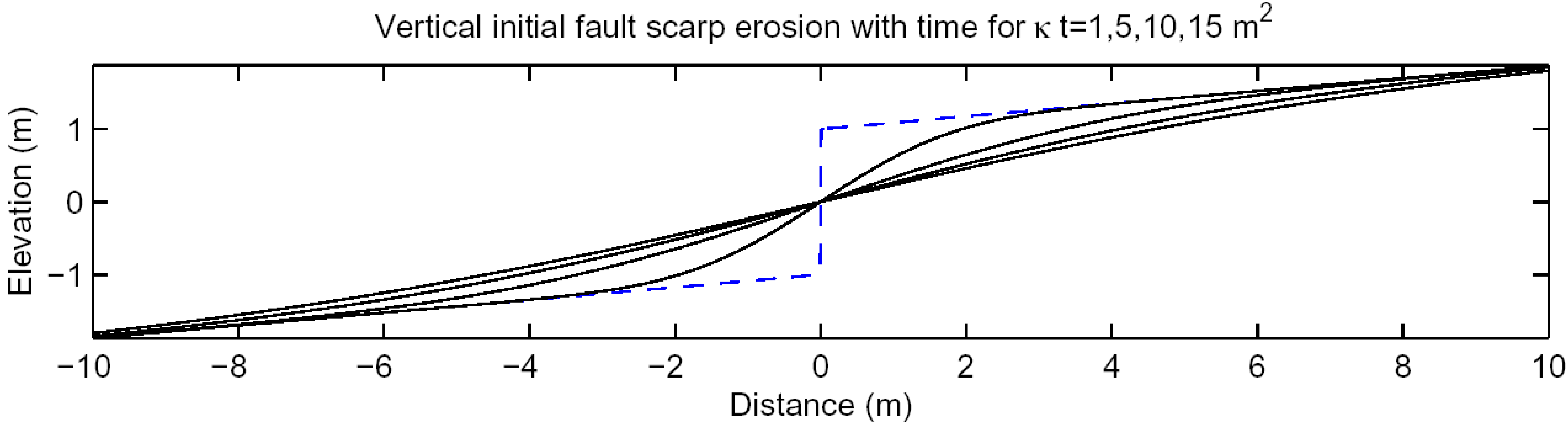
$\lim \Delta x \to dx, \Delta t \to dt$

$$\frac{dH}{dt} = k\frac{d\left(\frac{dH}{dx}\right)}{dx}$$

$$\frac{dH}{dt} = k\frac{d^2 H}{dx^2}$$

“diffusion” erosion

Simple scarp diffusion: Vertical initial form

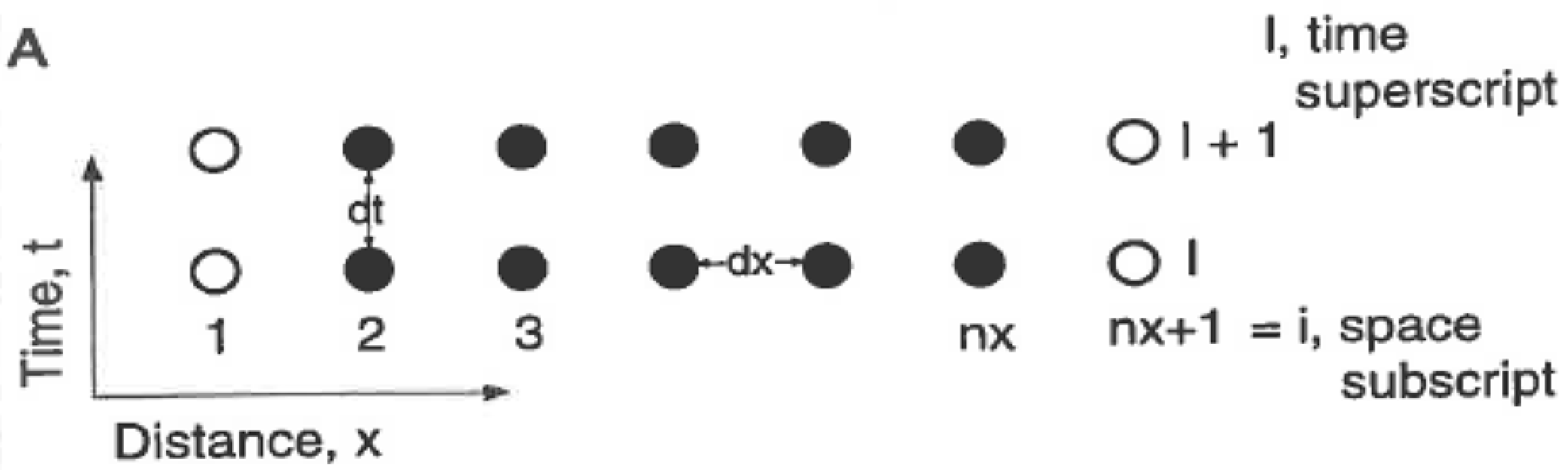


$$H(x,t) = a \operatorname{erf}\left(\frac{x}{2\sqrt{kt}}\right) + bx$$

B = “fan” slope   a = half-offset

kt = “morphologic age”

“analytic solution”

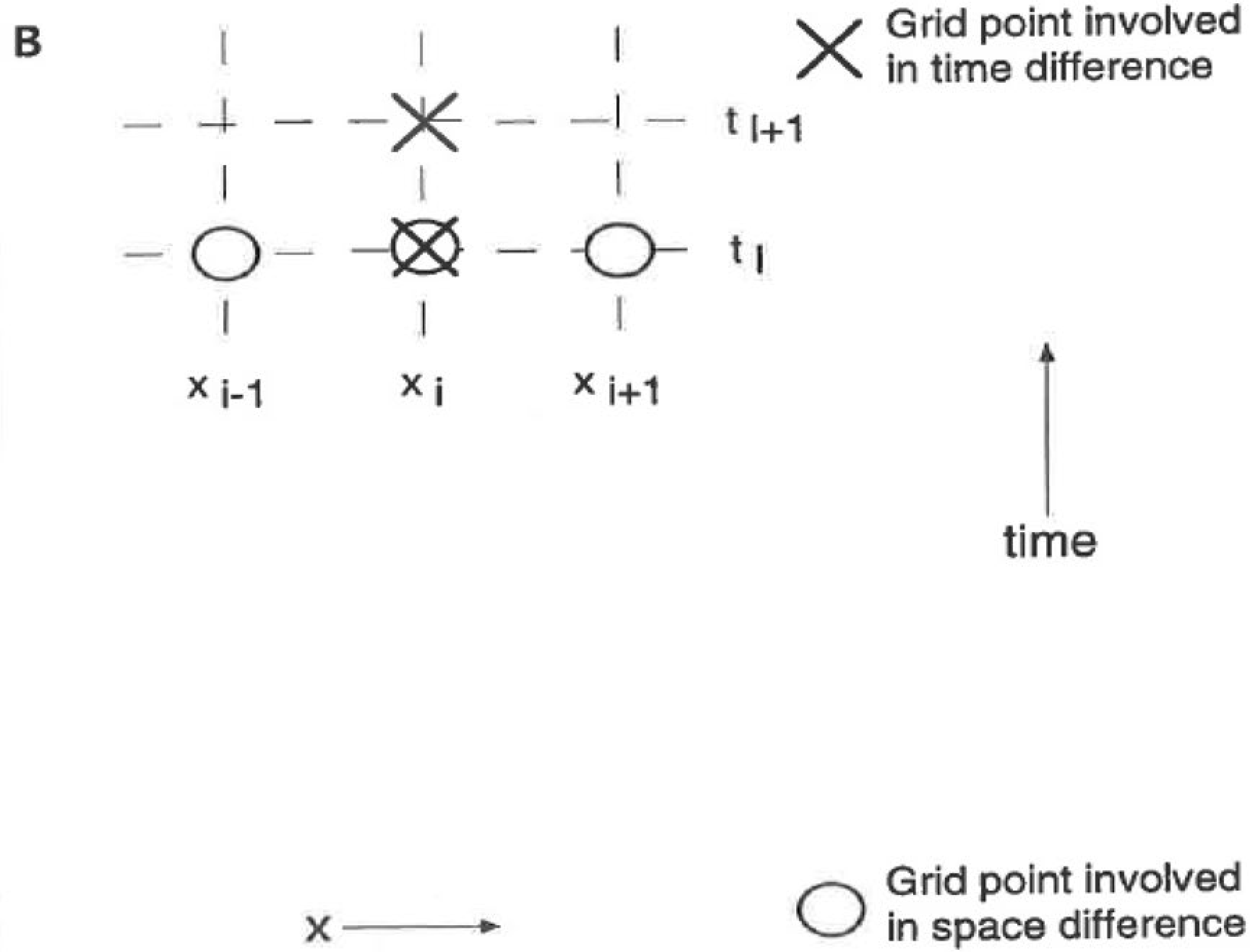


Explanation of nodes:

● interior node, elevation is determined by finite difference eqns.

○ exterior node or boundary condition, elevation is fixed

"numerical solution"


B
Grid point involved in time difference
t_{l+1}
t_l
x_{i-1}
x_i
x_{i+1}
time
x
Grid point involved in space difference

Recall the basic diffusion equation: $\frac{dH}{dt} = k\frac{d^2H}{dx^2}$ (1)

Use a forward difference to approximate the change in elevation with time: $\frac{dH}{dt} = \frac{H_i^{l+1} - H_i^l}{\Delta t}$ (2)

Recall that the right side of (1) is the difference in flux into and out of x_i:

$$q_{s\,out} - q_{s\,in} \tag{3}$$

$$k\left(\frac{H_{i+1}^l - H_i^l}{\Delta x}\right) - k\left(\frac{H_i^l - H_{i-1}^l}{\Delta x}\right) \tag{4}$$

$$\frac{k}{\Delta x^2}\left(H_{i+1}^l - 2H_i^l + H_{i-1}^l\right) \tag{5}$$

Then, combine substitute (2) and (5) into the left and right sides respectively of (1):

$$\frac{H_i^{l+1} - H_i^l}{\Delta t} = \frac{k}{\Delta x^2}\left(H_{i+1}^l - 2H_i^l + H_{i-1}^l\right) \tag{6}$$

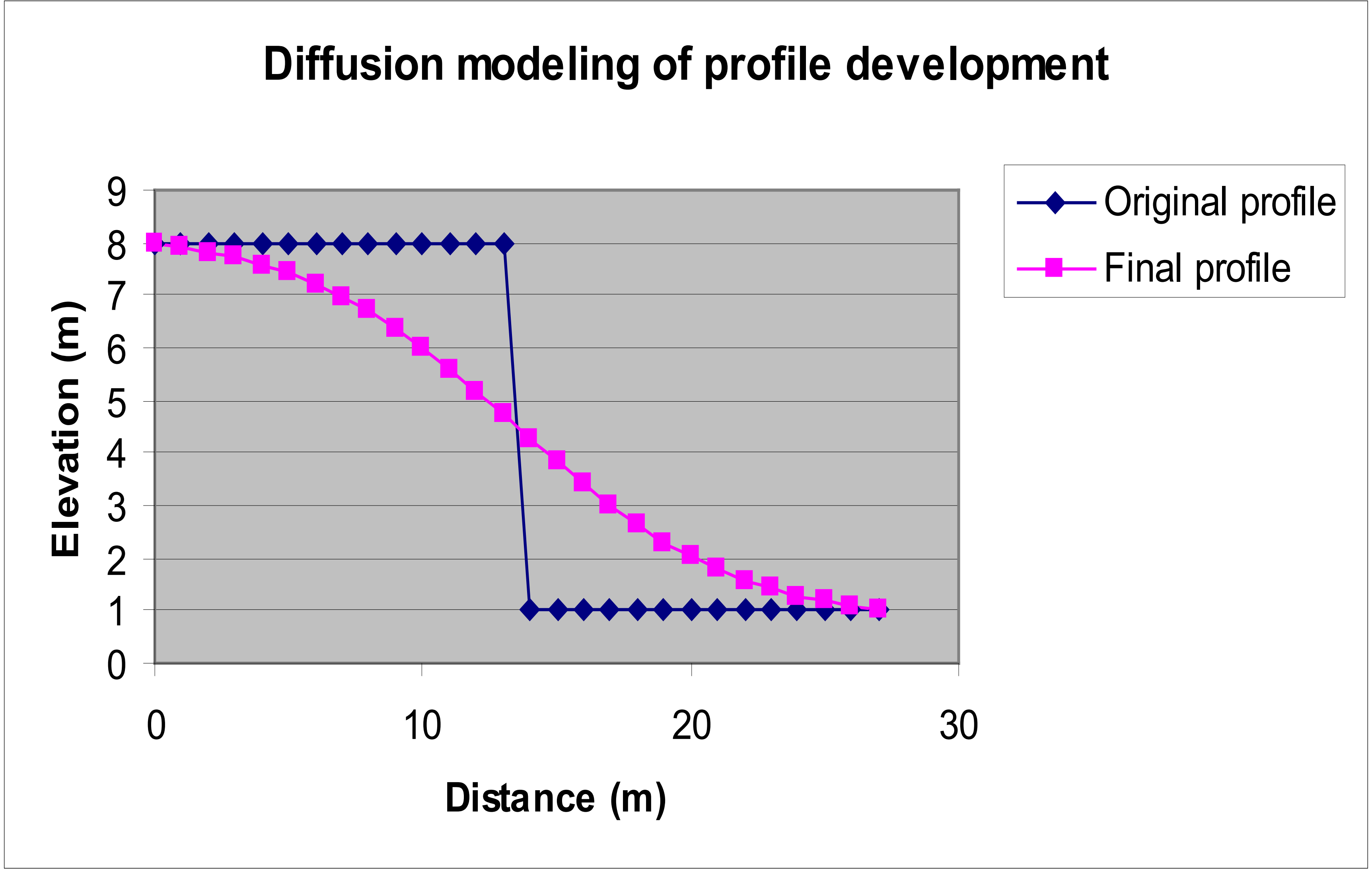
And rearrange:

$$H_i^{l+1} = H_i^l + \frac{k\Delta t}{\Delta x^2}\left(H_{i+1}^l - 2H_i^l + H_{i-1}^l\right) \tag{7}$$

$$H_i^{l+1} = H_i^l + \lambda\left(H_{i+1}^l - 2H_i^l + H_{i-1}^l\right) \tag{8}$$

Where $\lambda = \frac{k\Delta t}{\Delta x^2}$.

http://arrowsmith410-598.asu.edu/Lectures/Lecture8/ "numerical solution"



https://activetectonics.blogspot.com/2020/12/exploring-diffusion-for-hillslope.html

“numerical solution”