

Advanced Structural Geology, Fall 2022

Vectors and structural planes--I

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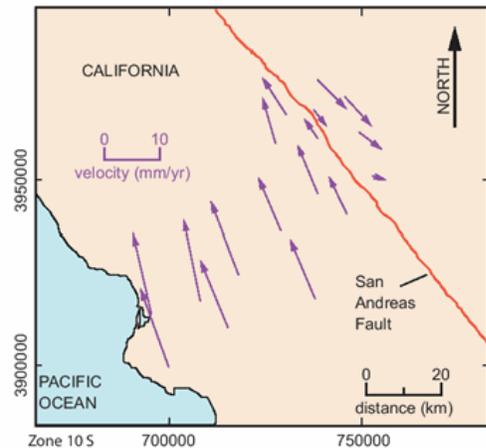
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Mostly from Chapters 1 and 7, Ragan, Structural Geology: An
Introduction to Geometrical Techniques

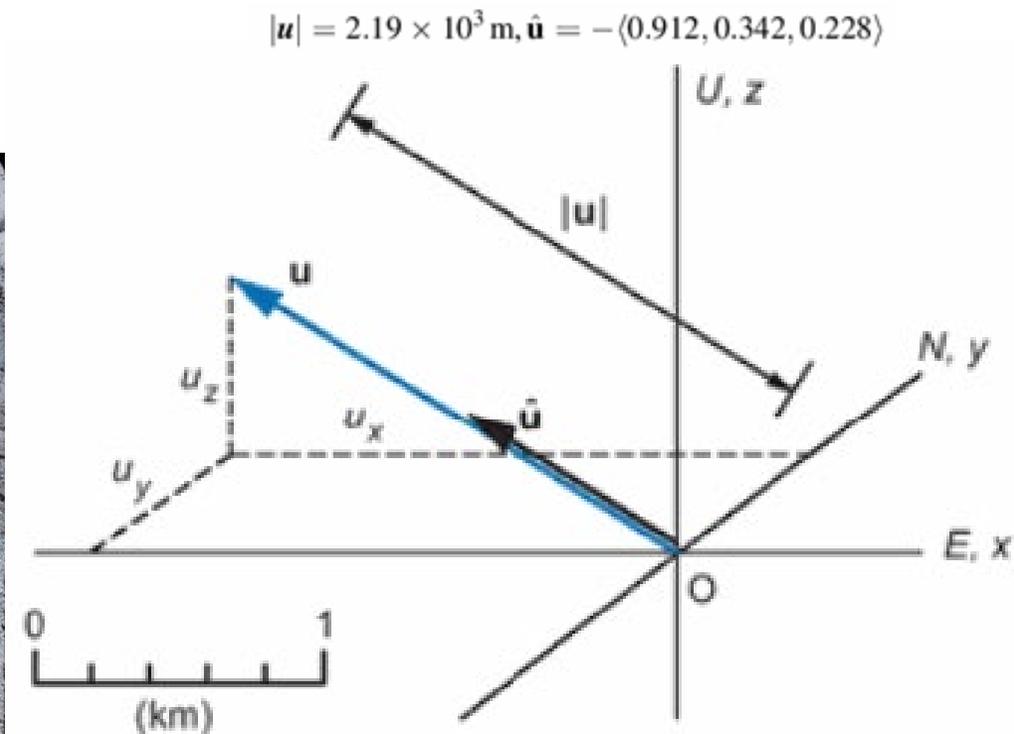


Vectors

- Axes have orientation but no sense. Lineations in metamorphic rocks, lines of intersection and poles to planes are examples
- Vectors have both orientation and sense. Examples include some linear sedimentary structures, paleomagnetic directions, displacements, DIP! TRACTIONS!



Vector field



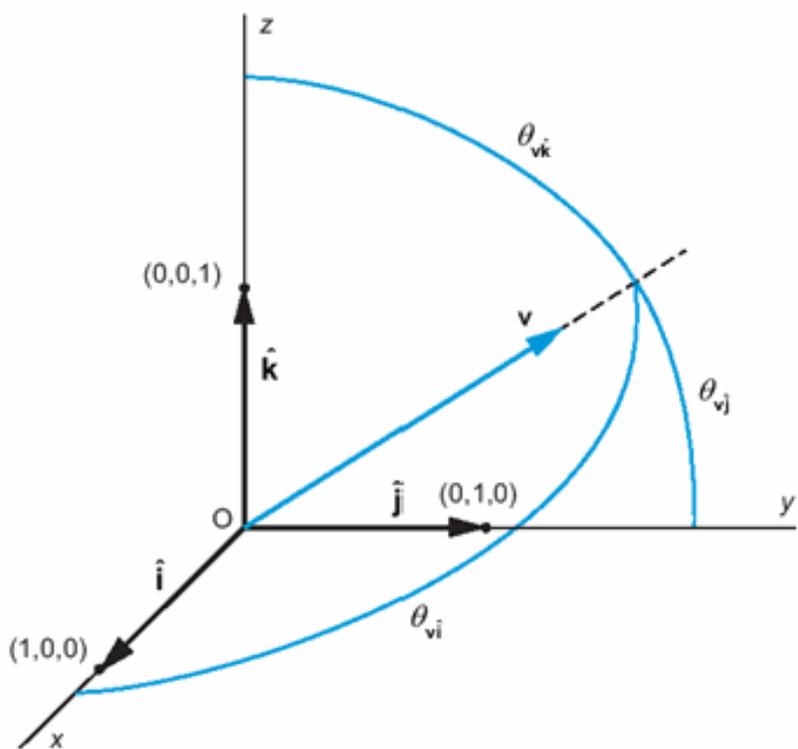


Figure 2.9 Right-handed Cartesian reference frame (x, y, z) , unit base vectors $(\hat{i}, \hat{j}, \text{ and } \hat{k})$, and direction angles $(\theta_{v\hat{i}}, \theta_{v\hat{j}}, \theta_{v\hat{k}})$ between the arbitrary vector, \mathbf{v} , and the base vectors. See Varberg et al. (2006) in Further Reading.

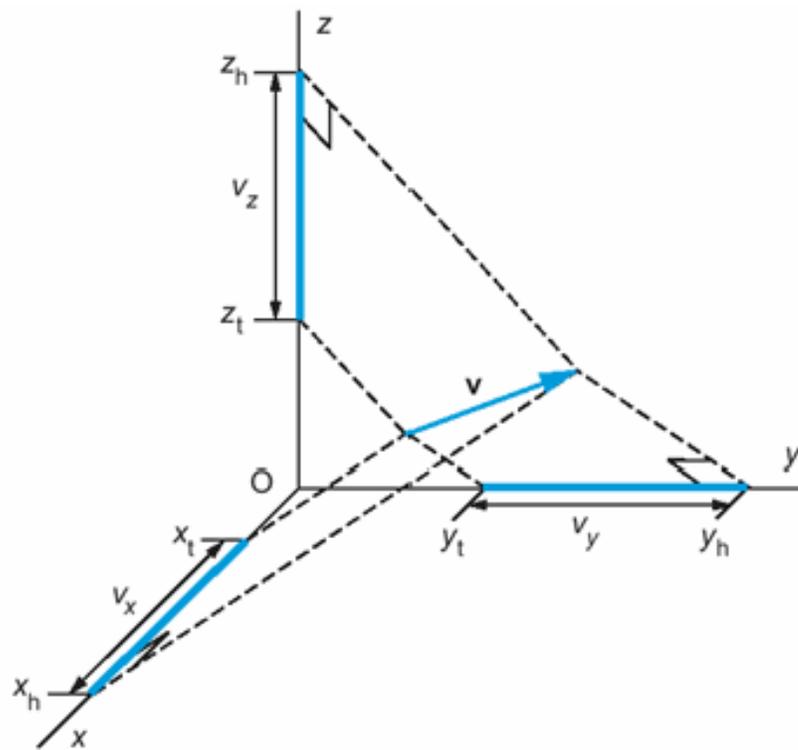


Figure 2.10 Projections of the arbitrary vector, \mathbf{v} , onto the Cartesian coordinate axes are the rectangular components, (v_x, v_y, v_z) . See Varberg et al. (2006) in Further Reading.

The vector, \mathbf{v} , may be written algebraically in either of two forms:

$$\mathbf{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} \text{ or } \mathbf{v} = \langle v_x, v_y, v_z \rangle \quad (2.2)$$

The first form is a vector equation utilizing the components to scale the base vectors. In the second form one reads $\langle \rangle$ as: “has the components” and it is understood that these components would be associated with the respective base vectors in the vector equation. Note that the components carry the dimensions and units of physical vectors, because the base vectors are dimensionless and unitless quantities.

The **vector magnitude** of an arbitrary vector, \mathbf{v} , is calculated in terms of its components as:

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad (2.3)$$

The **direction cosine** is the cosine of a direction angle, and it is defined as the ratio of the vector component to the vector magnitude:

$$\cos \theta_{v\hat{i}} = v_x/|\mathbf{v}|, \cos \theta_{v\hat{j}} = v_y/|\mathbf{v}|, \cos \theta_{v\hat{k}} = v_z/|\mathbf{v}| \quad (2.4)$$

The range of the direction cosines is $-1 \leq \cos \theta \leq +1$. Values outside this range would be erroneous. Given the vector magnitude and the direction in terms of the direction cosines, the **vector components** are:

$$v_x = |\mathbf{v}| \cos \theta_{v\hat{i}}, v_y = |\mathbf{v}| \cos \theta_{v\hat{j}}, v_z = |\mathbf{v}| \cos \theta_{v\hat{k}} \quad (2.5)$$

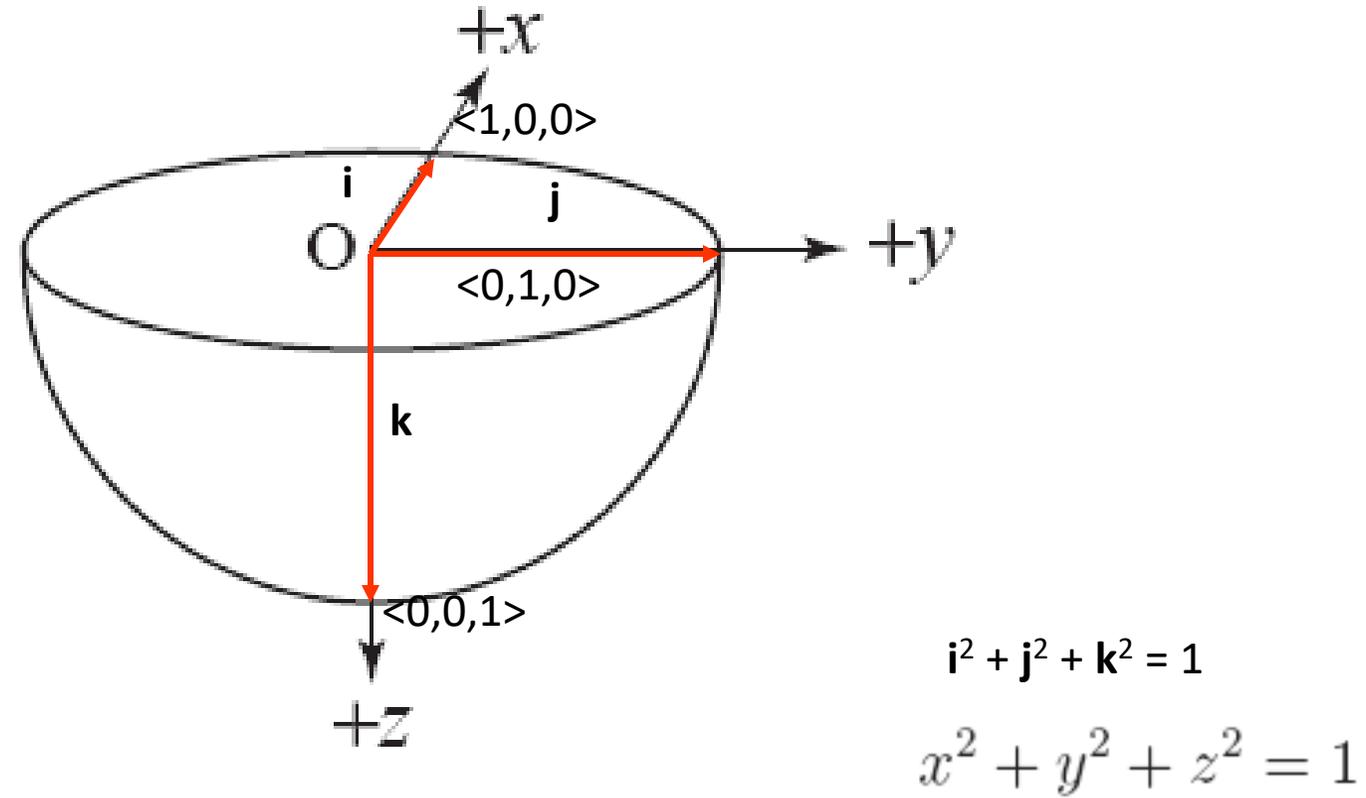


Figure 7.1: Lower hemisphere and the coordinate axes.

Any vector is the sum of its components in each of the coordinate directions

$$\mathbf{V} = V_x \mathbf{i} + V_y \mathbf{j} + V_z \mathbf{k}.$$

Unit base vectors and components

Length of vector $|\mathbf{V}| = V = \sqrt{V_x^2 + V_y^2 + V_z^2}. \quad (7.3)$

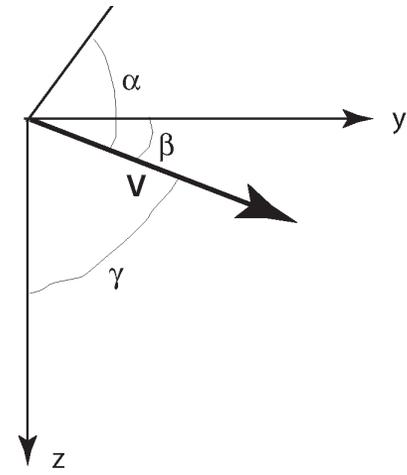
When used to represent the orientation of lines and poles we are interested in the directions not magnitudes of the associated vectors. It is then convenient to use only vectors of unit magnitudes. To find the unit vector with the same direction as a general vector, we *normalize* its components by dividing each by the magnitude. The three scalar components of this unit vector are called *direction cosines* and they are commonly given the symbols l , m and n , where

$$l = V_x/V, \quad m = V_y/V, \quad n = V_z/V. \quad (7.4)$$

$$l^2 + m^2 + n^2 = 1. \quad (7.5)$$

For use in plotting vectors as points on the stereonet *direction angles* are more useful and these are defined as

$$\alpha = \arccos l, \quad \beta = \arccos m, \quad \gamma = \arccos n. \quad (7.6)$$



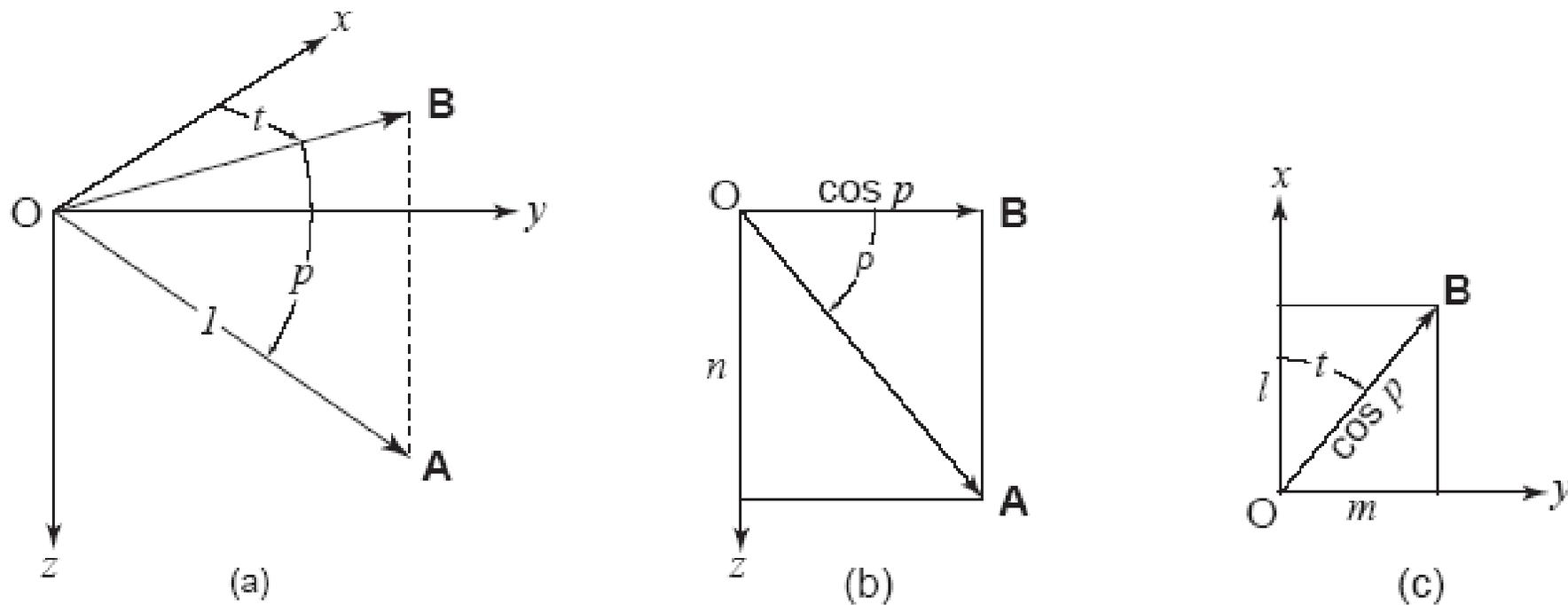


Figure 7.3: Direction cosines from plunge and trend.

the direction cosines of A are

$$l = \cos p \cos t, \quad m = \cos p \sin t, \quad n = \sin p. \quad (7.7)$$

These may also be converted back to plunge and trend with

$$p = \arcsin n, \quad t = \arctan m/l. \quad (7.8)$$

```

function [l,m,n] = plunge_trend_to_dir_cosines(plunge, trend)

%JRA 4/16/09
%assumes degrees!

n = sind(plunge);
l = cosd(plunge).*cosd(trend);
m = sind(trend).*cosd(plunge);

function [plunge, trend] = dir_cosines_to_plunge_trend(l, m, n)
%JRA 4/16/09

plunge_radians = asin(n);
plunge=degrees(plunge_radians);

trend_radians = atan2(m,l);
trend=degrees(trend_radians);
if trend<0
    trend=trend+360;
end

```

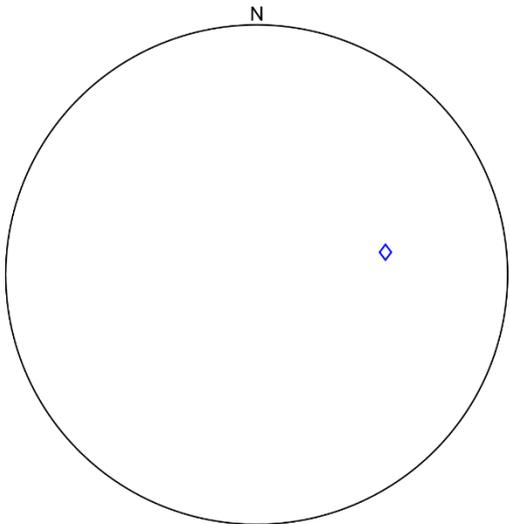
The *arctan* function on hand-held calculators and in most programming languages returns angles in the range $-90^\circ < t < 90^\circ$, that is, only trend angles in the NE and NW quadrants are reported. If $l < 0$ then the actual angle is in range $90^\circ < t < 270^\circ$ and it is necessary to add or subtract 180° to get the correct trend. In most programming languages and spreadsheet programs, the alternative function $\text{atan2}(m, l)$ gives the trend without need for this correction.

```
%input attitude:
plunge=35;
trend=080;
%~~~~~Plotting trend and
plunge~~~~~
plotdiamond(plunge,trend);

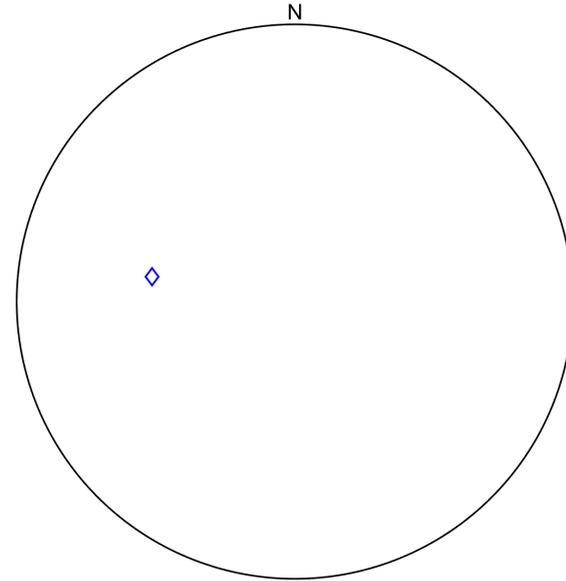
[l,m,n] =
plunge_trend_to_dir_cosines(plunge, trend);

fprintf('l = %0.2f, m = %0.2f, n =
%0.2f\n', l, m, n)
```

l = 0.14, m = 0.81, n = 0.57



```
%input attitude:
plunge=35;
trend=280;
l = 0.14, m = -0.81, n = 0.57
```



Sum of vectors

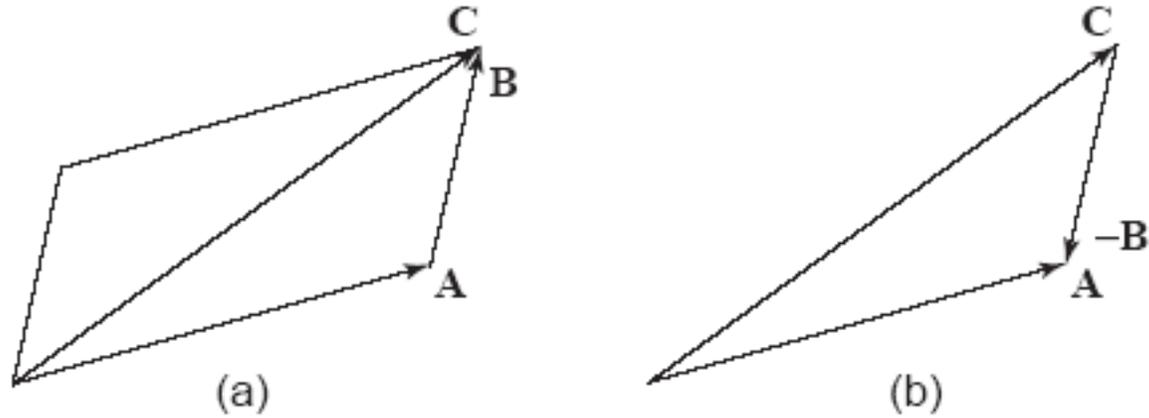
$$\mathbf{A} + \mathbf{B} = \mathbf{C}$$


Figure 7.4: Parallelogram rule: (a) addition; (b) subtraction.

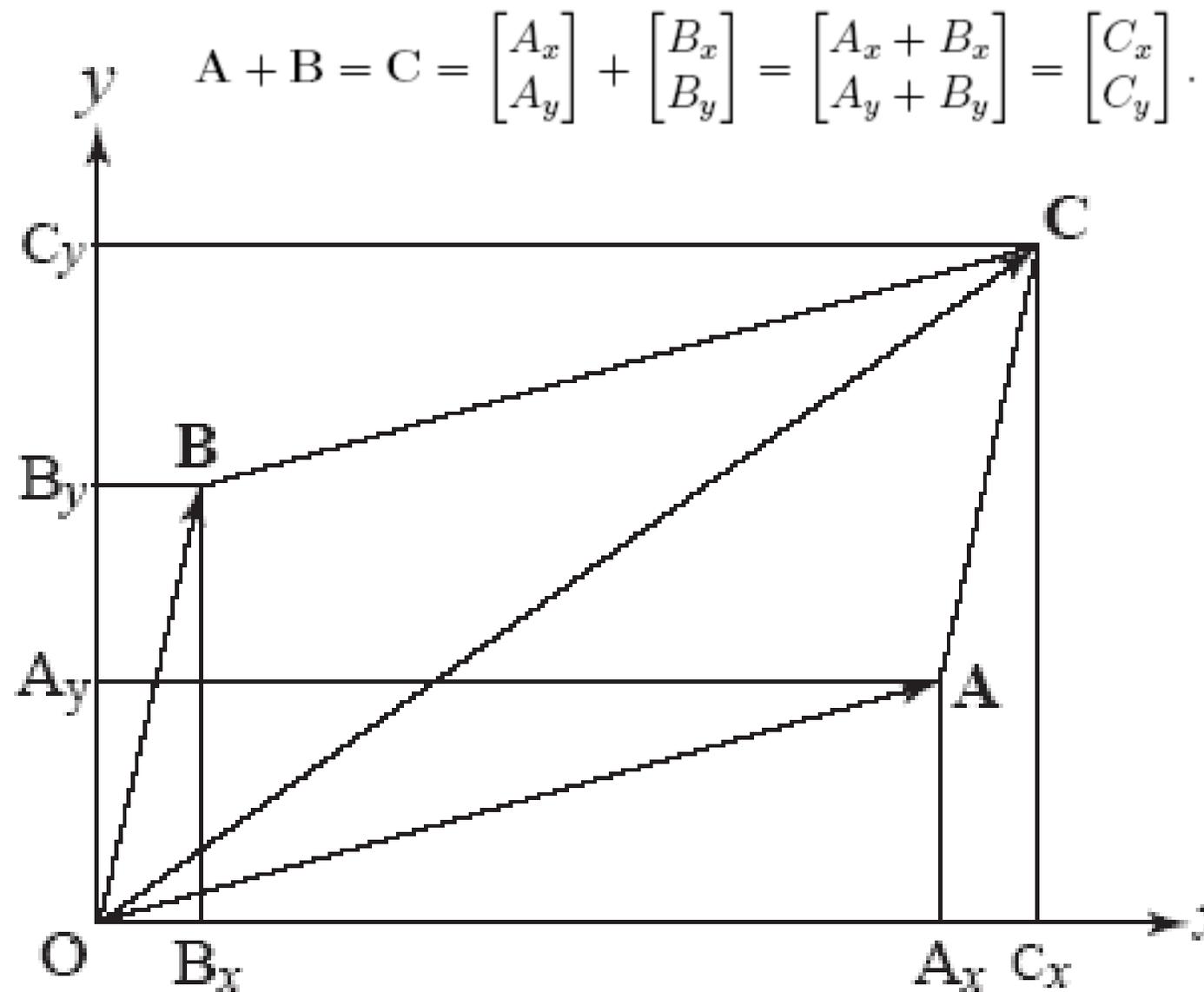


Figure 7.5: Vector addition using components.



Figure 2.12 Comb Ridge monocline near Bluff, Utah. The unit normal to bedding is \hat{n} and the base vector for the z-axis is \hat{k} . These are not drawn to scale. The rotation angle for the sedimentary bedding, is found using $\hat{n} \cdot \hat{k}$ to be $\theta_{\hat{n}\hat{k}} \approx 35^\circ$ based on the scalar product (2.7). Google Earth file: Figure 2.12 Comb Ridge UT monocline.kmz. UTM 12 S 617668.32 m E, 4124689.42 m N.

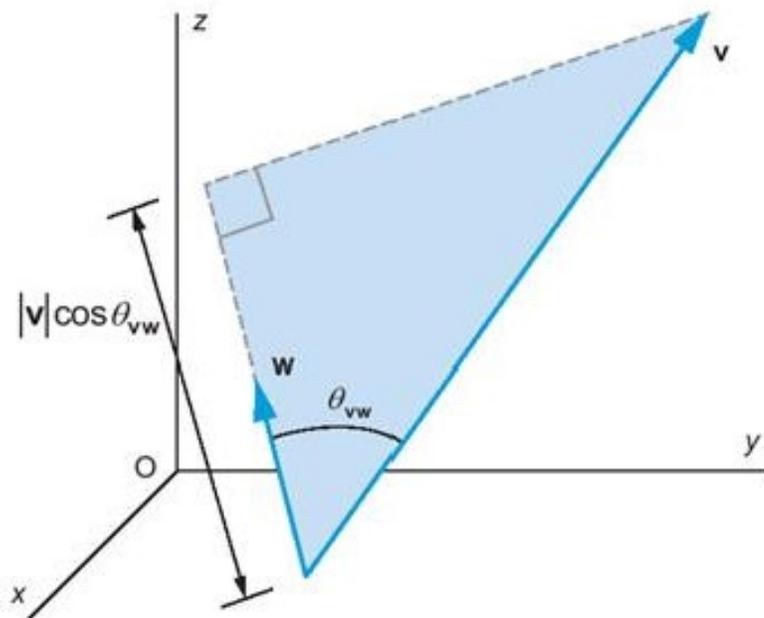


Figure 2.11 Geometric interpretation of the scalar product of the arbitrary vectors \mathbf{v} and \mathbf{w} . The direction angle between \mathbf{v} and \mathbf{w} is θ_{vw} . See Varberg et al. (2006) in Further Reading.

2.2.2 Scalar Product of Two Vectors

The multiplication of vectors may be accomplished in different ways and each of these has a distinct name and definition. The **scalar product** (or *dot product*) of two arbitrary vectors, \mathbf{v} and \mathbf{w} , is defined as the sum of the products of the respective components:

$$\mathbf{v} \cdot \mathbf{w} = v_x w_x + v_y w_y + v_z w_z \quad (2.6)$$

As the name implies, the result of this operation is a *scalar*, not a vector. Note that the order of multiplication does not change the result: $\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$.

While (2.6) serves to calculate the scalar product, an equivalent definition provides more geometric insight (Figure 2.11):

$$\mathbf{v} \cdot \mathbf{w} = |\mathbf{v}||\mathbf{w}| \cos \theta_{vw}, \quad 0 \leq \theta_{vw} \leq \pi \quad (2.7)$$

Note that $\cos \theta_{vw}$ is the direction cosine for the direction angle, which is the smaller angle between these vectors in the plane that contains them. You can slide \mathbf{w} so its tail coincides with the tail of \mathbf{v} to define this plane. The product $|\mathbf{v}| \cos \theta_{vw}$ is the orthogonal projection of \mathbf{v} onto the line parallel to \mathbf{w} . In other words, $|\mathbf{v}| \cos \theta_{vw}$ is the component of \mathbf{v} in the direction of \mathbf{w} . Therefore, the

Dot product

The first and simpler product of two vectors \mathbf{A} and \mathbf{B} is defined in terms of their magnitudes A and B and the angle ϕ between them as

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \phi, \quad (7.13)$$

where $0 \leq \phi \leq 90^\circ$. This has a useful geometrical interpretation: $B \cos \phi$ is the projection of \mathbf{B} onto \mathbf{A} and $A \cos \phi$ is the projection of \mathbf{A} onto \mathbf{B} . The dot product can also be expressed in component form as

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z. \quad (7.14)$$

$$\mathbf{A} \cdot \mathbf{B} = [A_x \quad A_y \quad A_z] \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = A_x B_x + A_y B_y + A_z B_z.$$

(Row times column multiplication)

If both vectors have unit magnitudes then Eqs. 7.13 and 7.14 combine to give the useful formula for finding the angle between them

$$\hat{\mathbf{u}}_1 \cdot \hat{\mathbf{u}}_2 = \cos \phi = l_1 l_2 + m_1 m_2 + n_1 n_2, \quad (7.16)$$

where (l_1, m_1, n_1) and (l_2, m_2, n_2) are the two sets of direction cosines. With this formula the angle between any two directions represented by unit vectors may be easily found. If $\phi = 90^\circ$, that is the two vectors are mutually perpendicular, this equation reduces to

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \quad (7.17)$$

and this can be used as a test for orthogonality.

ϕ is acute for $\mathbf{u} \cdot \mathbf{v} > 0$

ϕ is obtuse for $\mathbf{u} \cdot \mathbf{v} < 0$

$\phi = \pi/2$ for $\mathbf{u} \cdot \mathbf{v} = 0$

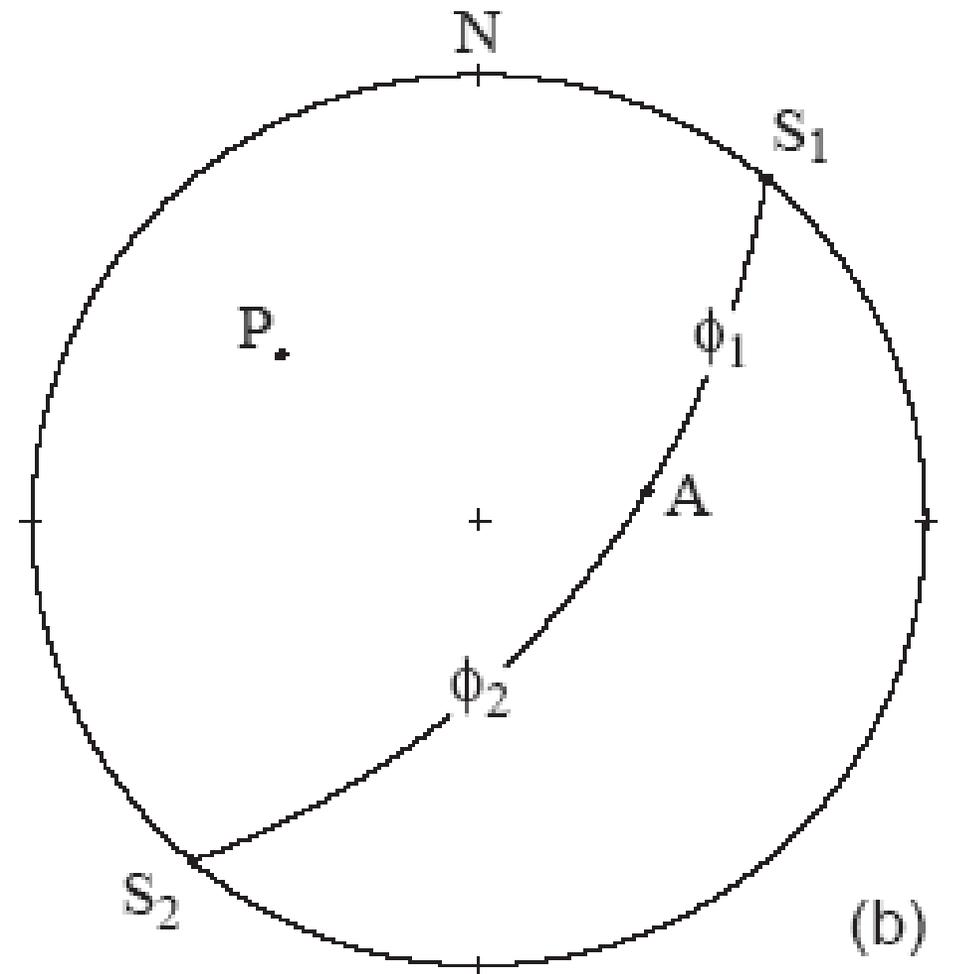
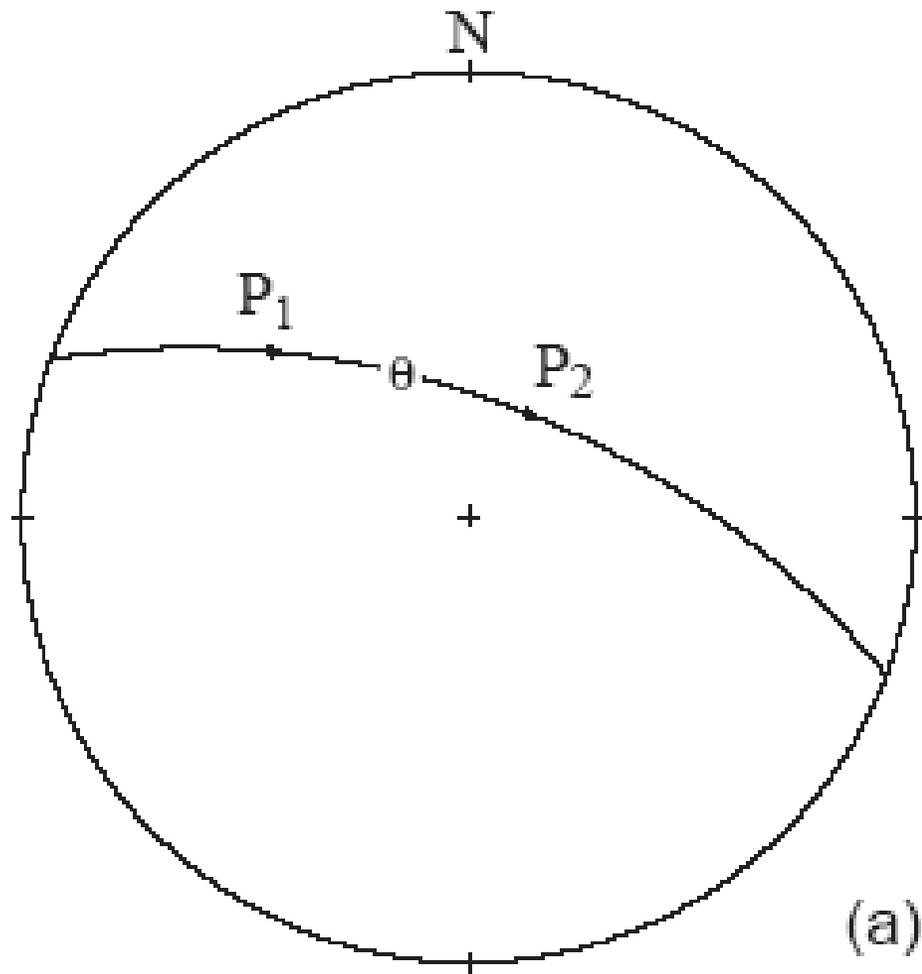


Figure 7.7: Dot product: (a) angle between vectors; (b) pitch of a line.
Or rake

Problem

- What is the angle between the pole vectors $P_1(30/310)$ and $P_2(60/030)$?

Solution

1. From each plunge and trend, using Eqs. 7.7, the direction cosines are

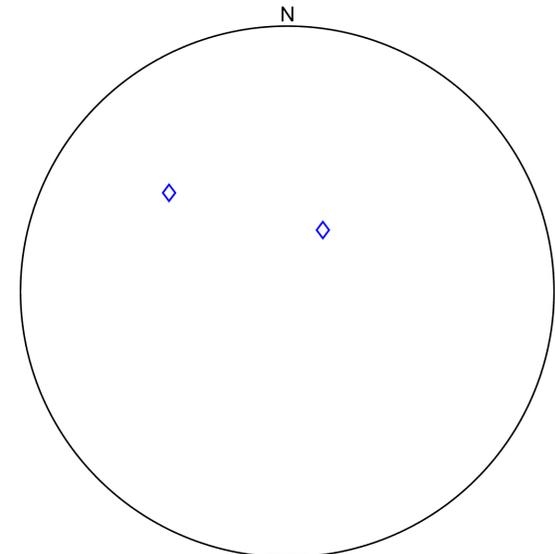
$$P_1(0.55667, -0.66341, 0.50000) \quad \text{and} \quad P_2(0.43301, 0.25000, 0.86603)$$

2. With Eq. 7.16 $\cos \phi = \frac{0.51503}{0.59820}$ or $\phi = 59^\circ$. This angle may be acute or obtuse; if acute, as here, it is the dihedral angle between the two planes (Fig. 7.7a).

```
[lone, mone, none] = plunge_trend_to_dir_cosines(30, 310);  
pone=[lone, mone, none];  
lone = 0.56, mone = -0.66, none = 0.50
```

```
[ltwo, mtwo, ntwo] = plunge_trend_to_dir_cosines(60, 030);  
ptwo=[ltwo, mtwo, ntwo];  
ltwo = 0.43, mtwo = 0.25, ntwo = 0.87
```

```
%two ways to do it:  
acosd(sum(pone.*ptwo));  
%or  
phi = acosd(dot(pone,ptwo));
```



In exactly the same way the angle between two lines L_1 and L_2 or between a line L and a pole P may also be found from the dot product.

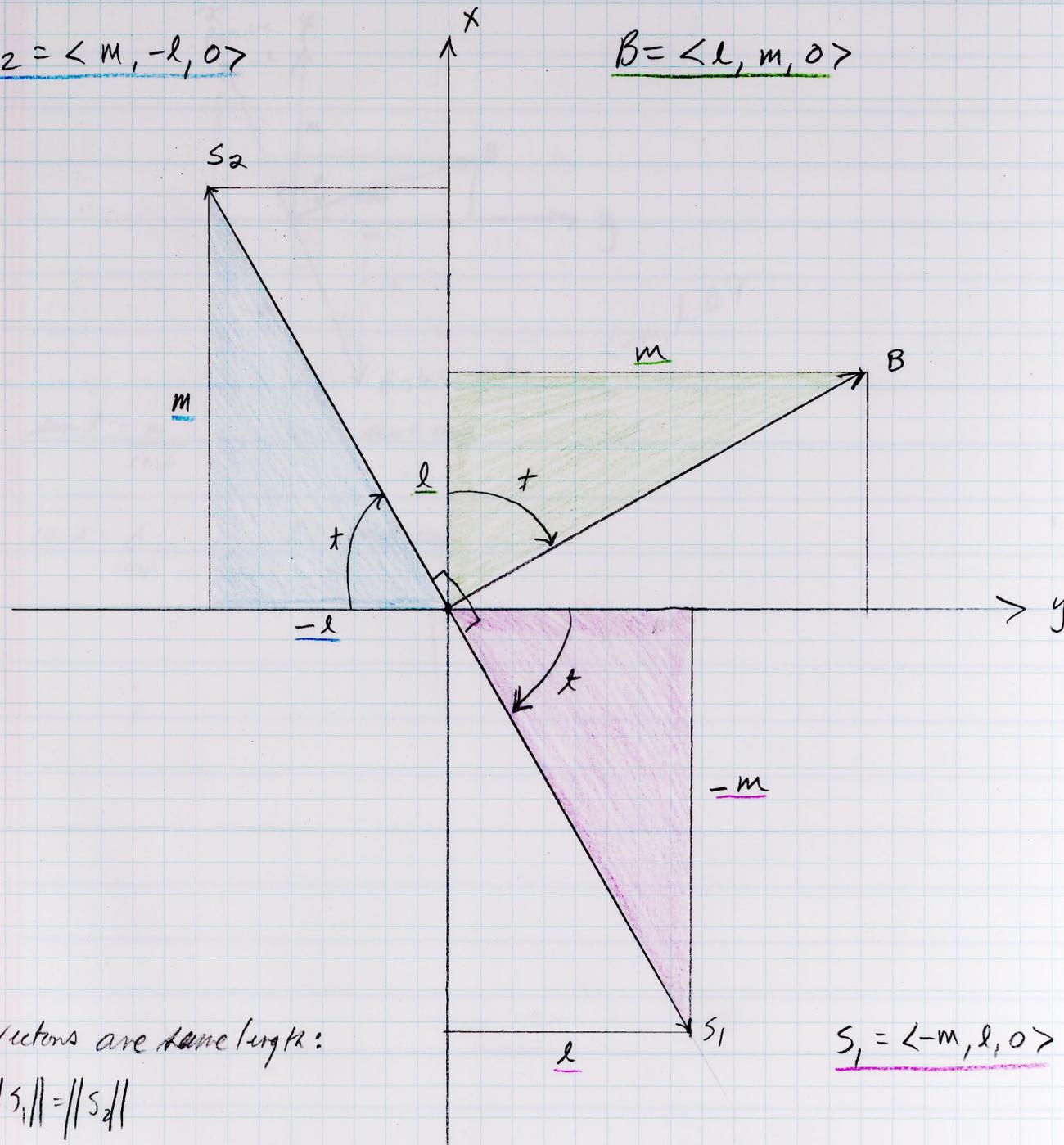
This technique can also be used to calculate the pitch of a line in a specified plane. There are several ways of doing this but the one which corresponds most closely with the previous graphical method requires the direction of the strike of the plane — it is perpendicular to the trend of the pole vector or equivalently, perpendicular to the trend of the dip vector. The orientation of this strike vector can be obtained in either of two ways.

1. From the trend t of either the pole or the dip vector, the strike direction is simply $t + 90^\circ$ or $t - 90^\circ$.
2. The direction of the strike may also be determined from the direction cosines (l, m, n) of either the pole or dip vector. From Eqs. 7.8, trend depends on m/l . The strike is then obtained from the negative reciprocal $-(l/m)$. This horizontal strike vector has two forms corresponding to its two equivalent ends

$$S_1(-m, l, 0) \quad \text{and} \quad S_2(m, -l, 0). \quad (7.19)$$

$$S_2 = \langle m, -l, 0 \rangle$$

$$B = \langle l, m, 0 \rangle$$



all 3 vectors are same length:

$$\|B\| = \|S_1\| = \|S_2\|$$

$$S_1 = \langle -m, l, 0 \rangle$$

```
[l, m, n] =plunge_trend_to_dir_cosines(30,310);
```

```
figure(3)  
clf  
primitivel;  
text(0,1, 'N', 'HorizontalAlignment', 'center',  
'VerticalAlignment', 'bottom')  
plotpointcolor(30,310,'b^');  
hold on
```

```
%first convert the pole to the two possible strike vectors:  
l1 = -m;  
m1 = l;  
n1 = 0;  
s1 = [l1, m1, n1];  
[plunge, trend] = dir_cosines_to_plunge_trend(l1, m1, n1);  
strike1 = trend;  
plotpointcolor(0,strike1,'rx');
```

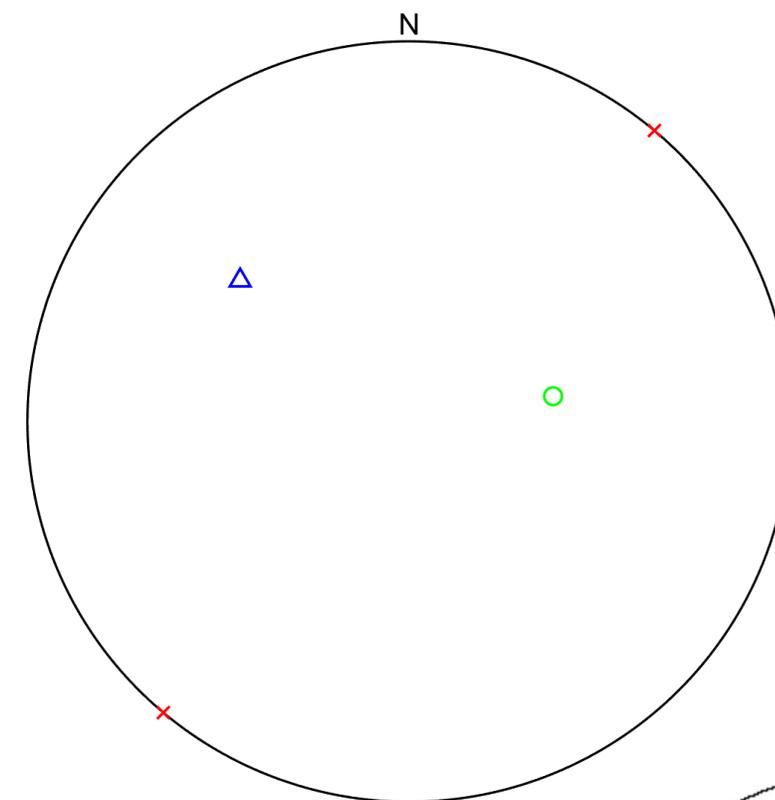
```
l2 = m;  
m2 = -l;  
n2 = 0;  
s2 = [l2, m2, n2];  
[plunge, trend] = dir_cosines_to_plunge_trend(l2, m2, n2);  
strike2 = trend;  
plotpointcolor(0,strike2,'rx');
```

```
%now get the direction cosines of A  
[la, ma, na] =plunge_trend_to_dir_cosines(48,080);  
A = [la, ma, na];  
plotpointcolor(48,080,'go');
```

```
%now do the dots  
phi1 = acosd(dot(s1,A))  
phi2 = acosd(dot(s2,A))
```

Problem

- The orientation of a plane is given by its pole vector $P(30/310)$. Determine the pitch of the apparent dip vector $A(48/080)$ (Fig. 7.7b).



$\phi_1 = 63.6463$

$\phi_2 = 116.3537$

