Advanced Structural Geology, Fall 2022

Vectors and structural planes--II

Ramón Arrowsmith

ramon.arrowsmith@asu.edu

Mostly from Chapters 1 and 7, Ragan, Structural Geology: An Introduction to Geometrical Techniques

School of Earth and Space Exploration
Arizona State University

Figure 2.13 Raplee anticline on the San Juan River near Mexican Hat, UT. Unit normal vectors $\hat{\mathbf{n}}$ on various sedimentary surfaces change orientation over the fold. Vectors **v** and **w** are parallel to a particular bedding surface and are used to define the normal vector to that surface using the vector product (2.10). Photograph by I. Mynatt. Google Earth file: Figure 2.13 Raplee Ridge UT anticline. kmz. UTM: 12 S 604042.03 m E, 4113941.08 m N.

2.2.3 Vector Product of Two Vectors

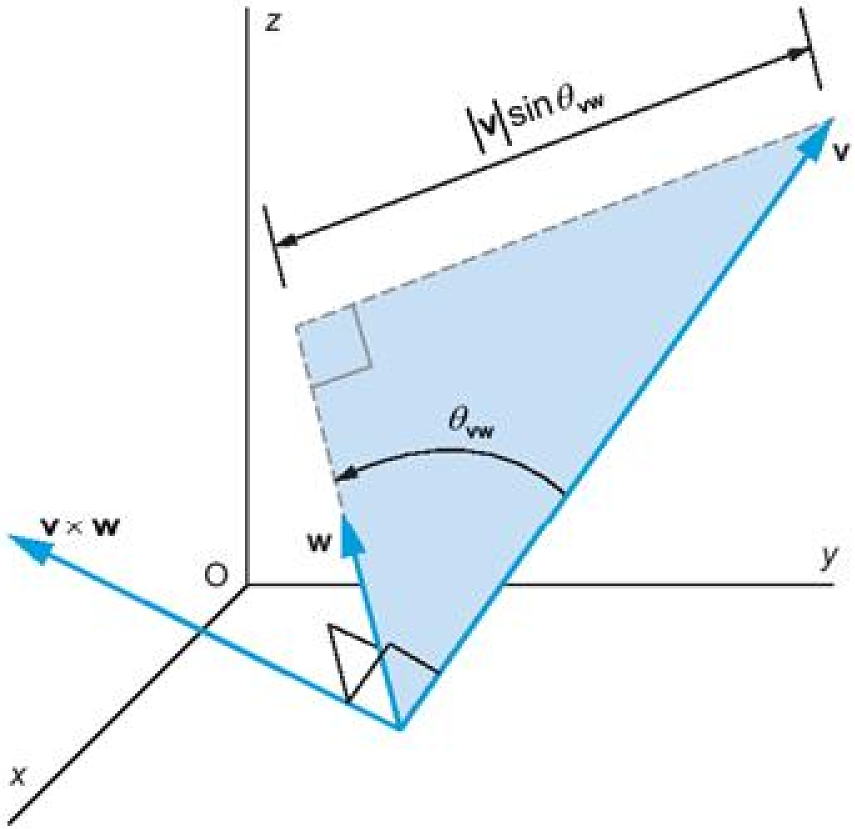
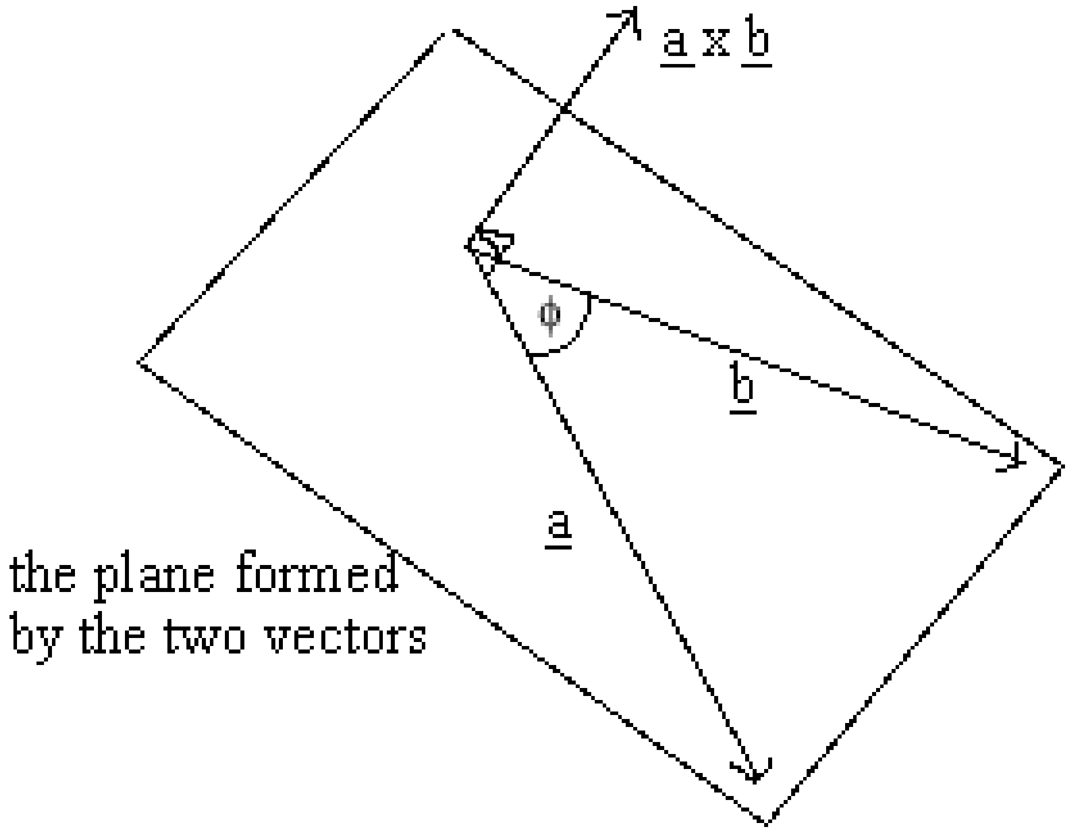


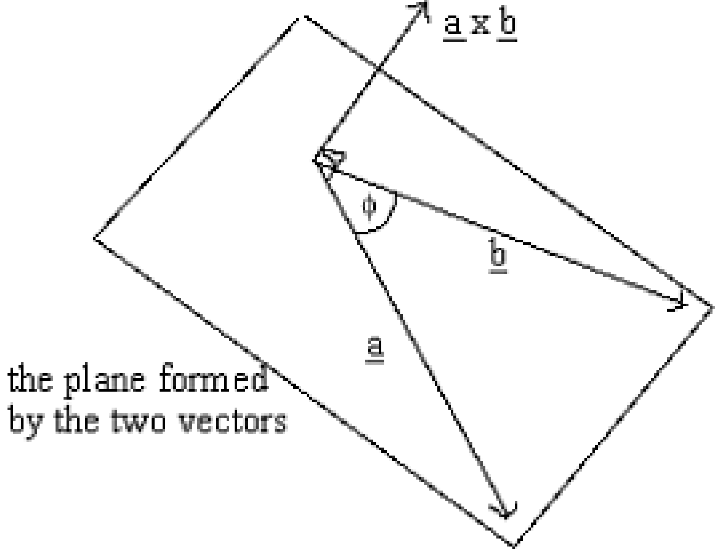
Figure 2.14 Geometric interpretation of the vector product of the arbitrary vectors **v** and **w**. The direction angle between **v** and **w** is $\theta_{\mathbf{vw}}$ and the direction of $\mathbf{v} \times \mathbf{w}$ is determined using the right-hand rule (see text). See Varberg et al. (2006) in Further Reading.

Cross Product

Sometimes we may want to construct a vector in 3 space that is perpendicular to any other 2 vectors (or, thus, to a plane).



http://members.tripod.com/~Paul_Kirby/vector/Vcrossproduct.html



$$\mathbf{C} = \mathbf{A} \times \mathbf{B}. \tag{7.20}$$

The product vector $\mathbf{C}$ is perpendicular to the plane of $\mathbf{A}$ and $\mathbf{B}$ and its direction is determined by the *right-hand rule:* if the fingers of the right hand point from $\mathbf{A}$ toward $\mathbf{B}$ through the smaller angle, the thumb points in the direction of $\mathbf{C}$. If the order is reversed, the direction of $\mathbf{C}$ is also reversed, hence the order does make a difference. This condition can be express as $\mathbf{A} \times \mathbf{B} = -(\mathbf{B} \times \mathbf{A})$. In other words, the cross product is not commutative.

The magnitude of the cross product vector is defined as

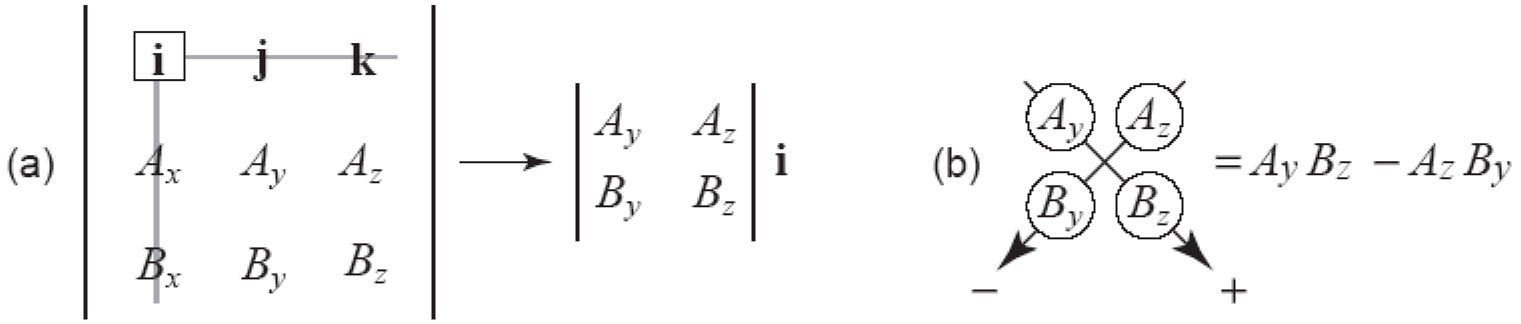
$$C = AB \sin\phi, \tag{7.21}$$

where, as before, ϕ is the smaller angle between the two vectors.

Solve as **determinant**

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}.$$

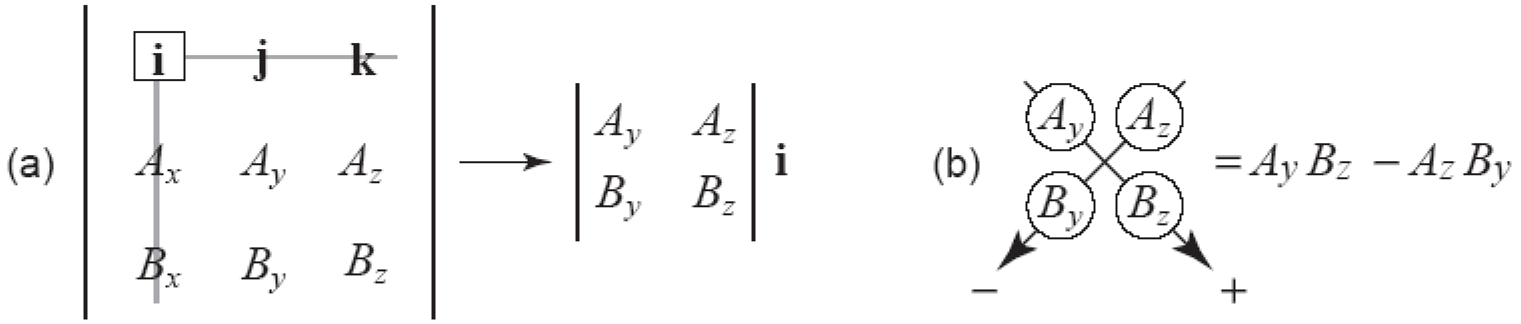
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \mathbf{i} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \mathbf{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \mathbf{k}.$$



$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y)\mathbf{i} - (A_x B_z - A_z B_x)\mathbf{j} + (A_x B_y - A_y B_x)\mathbf{k}. \tag{7.23}$$

Thus

$$C_x = A_y B_z - A_z B_y, \quad C_y = A_z B_x - A_x B_z, \quad C_z = A_x B_y - A_y B_x. \tag{7.24}$$



Problem 12: page 12 in Ragan chapter 7

Several important problems are easily solved using the cross product. The attitude of a plane, as represented by its pole vector $\mathbf{P}$, can be obtained directly from two apparent dip vectors $\mathbf{A}_1$ and $\mathbf{A}_2$. This is written as

$$\mathbf{P} = \mathbf{A}_1 \times \mathbf{A}_2. \tag{7.26}$$

Problem

- From apparent dip vectors $\mathbf{A}_1(20/286)$ and $\mathbf{A}_2(30/036)$ determine the attitude of the plane (Fig. 7.10a).

Solution

1. From the plunge and trend of each apparent dip vector, the two sets of direction cosines are

$$\mathbf{A}_1(0.30593, -0.88850, 0.34202) \quad \text{and} \quad \mathbf{A}_2(0.70063, 0.50904, 0.50000).$$

TYPO: 0.25899 -0.9032

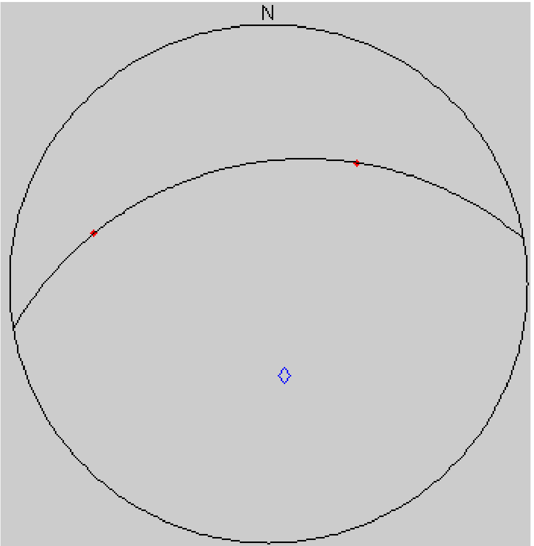
```
[lA1, mA1, nA1] =plunge_trend_to_dir_cosines(20,286);
A1=[lA1, mA1, nA1];

[lA2, mA2, nA2] =plunge_trend_to_dir_cosines(30,036);
A2=[lA2, mA2, nA2];

P = cross(A1,A2);
mag = sqrt(P(1)^2 + P(2)^2 + P(3)^2);
P(1) = P(1)/mag;
P(2) = P(2)/mag;
P(3) = P(3)/mag;

lp = P(1); mp = P(2); np = P(3);
[pole_plunge, pole_trend] = dir_cosines_to_plunge_trend(lp, mp, np);
%Dip vector from the pole should be
ld = -lp; md = -mp; nd = cosd(pole_plunge);
[dip, dipdir] = dir_cosines_to_plunge_trend(ld, md, nd);
```

A1 l = 0.2590 m = -0.9033 n = 0.3420
A2 l = 0.7006 m = 0.5090 n = 0.5000
P l = -0.6294 m = 0.1108 n = 0.7692
Pole plunge = 50.2787 and trend = 170.0190
dip = 39.7213 and dipdir = 350.0190



Problem 13: page 13 in Ragan chapter 7

$$\mathbf{I} = \mathbf{P}_1 \times \mathbf{P}_2. \tag{7.27}$$

Problem

- From two pole vectors $\mathbf{P}_1(70/146)$ and $\mathbf{P}_2(50/262)$ determine the line of intersection of the two planes (Fig. 7.10b).

Solution

1. The components are $\mathbf{P}_1(-0.28355, 0.19126, 0.93969)$ and $\mathbf{P}_2(-0.08946, -0.63653, 0.76604)$
2. The normalized components of the intersection vector are $\mathbf{I}(0.95243, -0.17030, 0.25273)$.

Answer

- The attitude of the line of intersection is $\mathbf{I}(15/010)$.

```
lP1, mP1, nP1] =plunge_trend_to_dir_cosines(70,146);
P1=[lP1, mP1, nP1];
ld1 = -lP1; md1 = -mP1; nd1 = cosd(70);
[dip, dipdir] = dir_cosines_to_plunge_trend(ld1, md1, nd1);

[lP2, mP2, nP2] =plunge_trend_to_dir_cosines(50,262);
P2=[lP2, mP2, nP2];
ld2 = -lP2; md2 = -mP2; nd2 = cosd(50);
[dip, dipdir] = dir_cosines_to_plunge_trend(ld2, md2, nd2);

I = cross(P1,P2);
%need to renormalize!!!
mag = sqrt(I(1)^2 + I(2)^2 + I(3)^2);
I(1) = I(1)/mag;
I(2) = I(2)/mag;
I(3) = I(3)/mag;

li = I(1); mi = I(2); ni = I(3);
[plunge, trend] = dir_cosines_to_plunge_trend(li, mi, ni);
```

P1 l = -0.2835 m = 0.1913 n = 0.9397

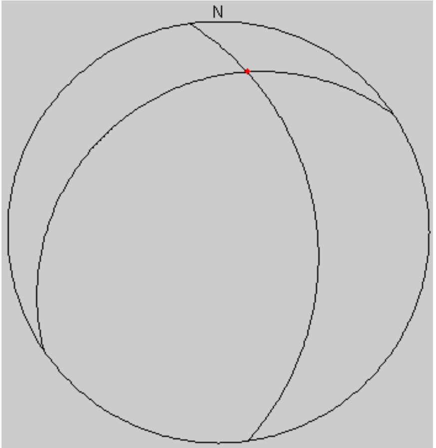
dip = 20.0000 and dipdir = 326.0000

P2 l = -0.0895 m = -0.6365 n = 0.7660

dip = 40.0000 and dipdir = 82.0000

I l = 0.9524 m = 0.1703 n = 0.2527

Intersection plunge = 14.6392 and trend = 10.1375



Practice

1.8 TRUE DIP & STRIKE

In some field situations it may not be possible to measure the true dip and strike directly. However, if apparent dips in two different directions are known, the attitude of the plane can be determined.

Problem

- From the two apparent dips 20/296 and 30/046 determine the true dip and strike of the plane.

Determine strike and dip

	l	Δh	t
P_1	983.3 m	−24.7 m	23.8°
P_2	1563.6 m	−48.3 m	76.4°

Table 1.1: Data for the three-point problem.

t is trend from base point o to points

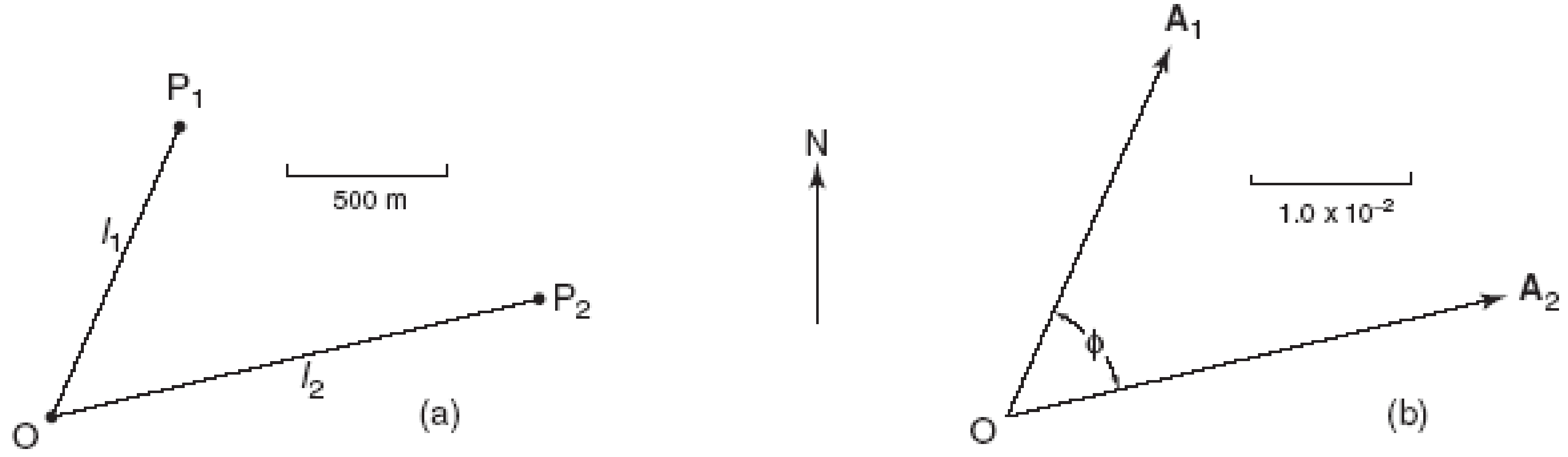


Figure 1.18: Three-point problem: (a) map of surveyed points; (b) apparent dip vectors.