Chapter 4

PLANES & TOPOGRAPHY

4.1 EXPOSURES ON HORIZONTAL SURFACES

In Chapter 2 the simplest examples of the determination of thickness assumed that the earth's surface was a horizontal, geometrically perfect plane. The intersection of inclined layers with this surface results in an *outcrop pattern*. Represented in map view this pattern is a simple geologic map.

In this case the width of the outcrop bands depends on two factors: the actual *thickness* of the layers and the angle of *dip* of each layer. The separate effect of each of these factors is shown in Fig. 4.1. In essence, these same relationships also apply to less-than-perfect real horizontal topographic surfaces.

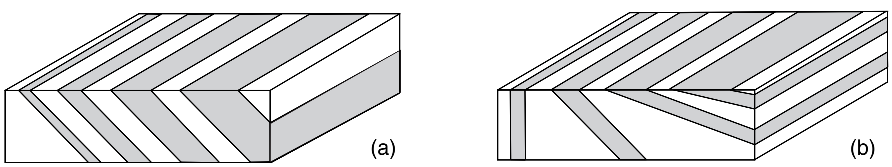

Figure 4.1: Outcrop width: (a) varies with thickness; (b) varies with dip.

In the special case of a vertical layer, the outcrop width in map view is equal to the thickness of the layer. This unique relationship results from the fact that the map shows such a layer in edge view, that is, a line of sight in viewing the map coincides with a line which is parallel to the vertical layer. In estimating thickness of tabular objects, one instinctively seeks just such a view.

In the more general case of an inclined layer, one line of sight is always identifiable on a geologic map; it is in the dip direction. For layers inclined an auxiliary view perpendicular to this line could easily be constructed that would show the layers in edge view and therefore in true thickness (Fig. 4.2a).

However, it is unnecessary to make this construction because the same information can be obtained directly from the map itself. Simply rotate the geologic map so that the dip direction is "north" and then view the map pattern along a line of sight inclined to the plane of the map at the dip angle. In this view the outcrop width, which is always greater than the thickness, is foreshortened by just the right amount to appear as true thickness (Fig. 4.2b). In adopting this oblique, down-dip view of the map it may help to reduce your depth perception by closing one eye. Clearly, this method of viewing the outcrop bands of inclined layers is limited to cases involving significant dip angles, for it is physically impossible to view horizontal strata in edge view along any line of sight of a map.

This principle is used in reverse for traffic signs painted on streets. By purposely distorting the letters as viewed vertically (map view) the foreshortening which accompanies the driver's oblique view of the road surface exactly compensates for the distortion and the warnings appear in normal proportions and are perfectly readable (see Fig. 4.3).

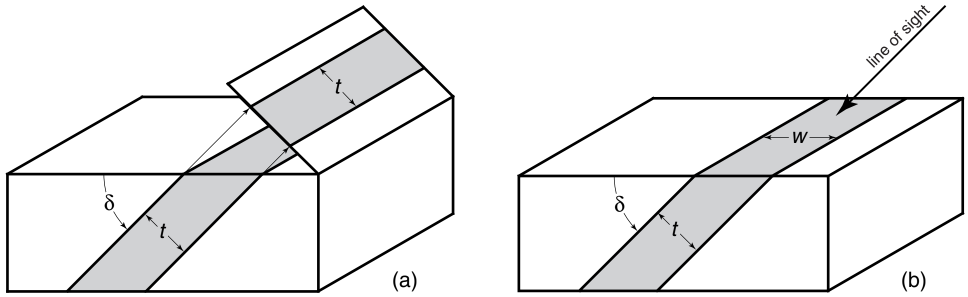

Figure 4.2: True thickness: (a) plane normal to dip line; (b) down-dip view.

Figure 4.3: View from the right along a line inclined to the page.

In effect, twisting such simple geologic maps so that the inclined strata are viewed in the down-dip direction restores the sedimentary beds to their original horizontal attitude. Contacts on the map then cease to be just lines separating stratigraphic units on the earth's surface; they can be seen to come to life as the depositional and erosional surfaces which they once were. A map viewed down-dip represents a kind of cross section, such as might be seen in the walls of the Grand Canyon. As such, the important dimension of sequence of deposition in time is added to the map. Unconformities become buried landscapes, and this view facilitates comparisons with the present earth's surface and the erosional processes responsible for its form. Certainly the possibility of completely overturned beds should be recognized, especially in areas of complex structure. In such cases, the down-dip view yields a picture of the strata which is upside-down, but such a view may actually help the interpretation of overturning if other obvious evidence is lacking.

4.2 EFFECT OF TOPOGRAPHY

In areas of sloping terrain, additional factors are involved in determining the character of outcrop patterns, and these include topographic slope angle and direction relative to the attitude of the strata, and on variations in slope angle and direction. In other words, in addition to thickness and dip, the map pattern also depends on the details of the topography. The relationships between dip and topography have been formalized into a series of statements, collectively called the *Rule of Vs*, by which the direction of dip can be estimated directly from the outcrop patterns. Wherever the trace of a plane crosses a valley, the resulting pattern is characteristic of the attitude of the plane. There are several distinct types of patterns.

1. *Horizontal planes:* Topographic contour lines can be thought of as the surface traces of imaginary horizontal planes. The outcrop traces of real horizontal planes therefore exactly follow the topographic contours. Such patterns are completely controlled by the topography; the outcrop trace faithfully reflects the local contour lines in every detail. Therefore, the outcrop pattern Vs upstream, just as the contour lines do (Fig. 4.4a).

2. *Planes inclined upstream:* As the attitude departs from the horizontal, with the dip direction in the upstream direction, the pattern made by the traces of the structural planes is progressively modified

into a blunter V, still pointing upstream (Fig. 4.4b). With steepening dip, the outcrop pattern is an increasingly subdued reflection of topographic detail.

3. *Vertical planes:* In the special case of a 90° dip, the outcrop traces are straight and parallel to the strike direction, regardless of topographic detail. There is no V at all, and thus no control on the pattern by the topography (Fig. 4.4c).

4. *Planes inclined downstream:* There are two general cases and one special boundary case.

   (a) With dip greater than valley gradient, the pattern Vs downstream (Fig. 4.5a).

   (b) If the dip angle and valley gradient are exactly equal, the outcrop trace will not cross the valley axis, and there is no V (Fig. 4.5b). However, streams generally steepen headward and a continuous planar structure will therefore cross somewhere upstream.

   (c) If the dip is less than the valley gradient, but still in a downstream direction, the pattern will V upstream (Fig. 4.5c).

(a) (b) (c)

Figure 4.4: Rule of Vs: (a) horizontal layer; (b) layer dipping upstream; (c) vertical layer.

As stated, these rules assume that the strike direction is at a right angle to the valley axis. The result is that the V patterns are approximately symmetrical. With other strike directions, asymmetrical Vs are produced, but in essence the rule still applies. In the limiting case when the valley and strike are parallel there is no V at all.

There is a simple, easily remembered statement which summarizes all these relationships: *the V of the outcrop trace points in the direction in which the formation underlies of the stream* (Screven, 1963).

Better yet, however, is to visualize the geometrical relationship between the structural planes and topography in three dimensions. In an area of topographic relief the outcrop pattern of uniformly dipping beds is irregular, yet if these same beds were viewed from an airplane in an oblique, down-dip direction, they would appear in edge view (the block diagram of Fig. 4.4b is very nearly in this orientation). The irregularities due to the topography are then eliminated and the traces of the inclined planes are straight; true thickness appears directly. This same relation would, of course, hold true for a scaled topographic relief model with the outcrop pattern included. By perceiving the earth's surface depicted by the topographic contour lines on the map as a relief model, the mind's eye can accommodate the influence of the topography of the outcrop

(a) (b) (c)

Figure 4.5: Rule of Vs: (a) layer dipping downstream; (b) layer and valley axis with equal slopes; (c) layer dipping downstream at an angle less than valley gradient.

pattern. This technique takes some effort to learn and practice is the key. Once the ability is attained, however, it is an enormously powerful aid in map interpretation, for even in areas of considerable and varied relief, and therefore of highly irregular map patterns, the structure can be viewed on the map in a down-dip direction with a great conceptual simplification.

4.3 DIP & STRIKE FROM A GEOLOGIC MAP

Previous examples we treated the attitude of inclined planes in semi-quantitative terms only. However, the actual dip and strike can be found if the spatial locations of three points on the plane are known. In the simplest case, two of the points with the same elevation can often be recognized.

Problem

- In Fig. 4.6, the trace of the lower bounding plane of the inclined layer cuts the topographic contours at points A, B and C with elevations $h_A = h_B = 620$ m and $h_C = 610$ m. What is the dip and strike?.

Solution

1. Draw line AB connecting the two points of equal elevation. This is, by definition, a line of strike.

2. Draw a perpendicular line from AB to point C. This is the true dip direction. The true dip angle is measured in the vertical section containing this line.

3. Draw a line parallel to this dip direction as a horizontal FL. Extend the strike line AB and draw a second strike line through C to intersect this FL.

4. These two points on FL represent the map locations of the line AB and the point C.

5. At a vertical distance below the map point C of $\Delta h = h_A - h_C = 10$ m plot the actual outcrop point C using the map scale. The inclined line of dip can be drawn and the dip angle measured.

Answer

- The attitude of the true dip is **D**(15/270).

The dip angle can also be found from the map distance D from the strike line AB to C and the vertical distance Δh using

$$\tan \delta = \Delta h / D. \tag{4.1}$$

Either graphically or analytically, choosing as widely spaced points as possible improves accuracy.

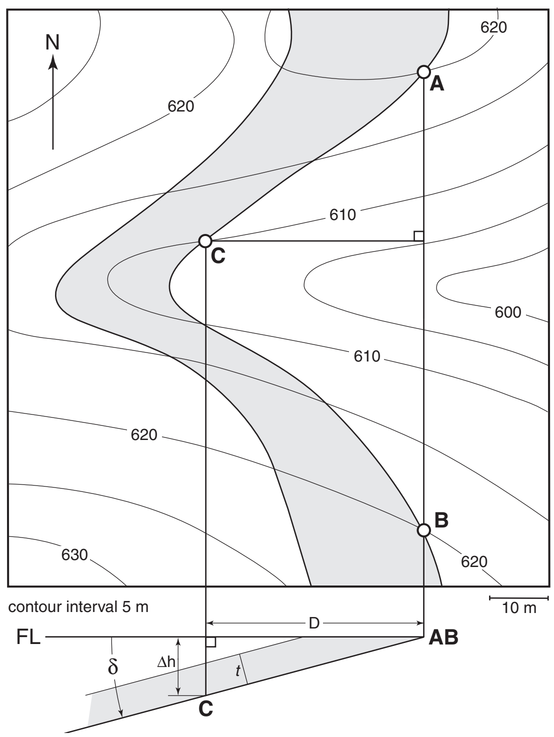

Figure 4.6: Dip and strike from the outcrop pattern.

4.4 LINEAR INTERPOLATION

More generally, the three points will have different elevations. We then need a way of locating a point with specified elevation lying on a line between two known ends. This involves *linear interpolation* and there are two complementary graphical approaches. The first uses previously established methods, while the second is simpler.

Problem

- Points O and A have elevations $h_O = 296$ m and $h_A = 178$ m. Map distance $D_{OA} = 300$ m. Locate point B on OA with elevation $h_B = 225$ m (Fig. 4.7a).

Construction I

1. With horizontal line OA as FL, construction a section showing a vertical line directly below surface point A (Fig. 4.7a). Using the map scale locate two points on this line.
   (a) Point X at a depth $\Delta h_A = (h_O - h_A) = (296 - 178) = 118$ m.
   (b) A point at an intermediate depth $\Delta h_B = (h_O - h_B) = (296 - 225) = 71$ m.
2. Draw line OX to represents the inclination of the line between map points O and A.
3. A horizontal line from the intermediate point intersects this inclined line OX at Y.
4. Project Y vertically back to OA to locate point B on the map with the required elevation.

This construction is based on the fact that right triangles OAX and OBY the *similar*les and a property of such triangles is that the lengths of corresponding pairs of sides have the same ratios—as Y divides AX in the ratio $\Delta h_B/\Delta h_A = 71/118$, so too B divides OA in this same ratio.

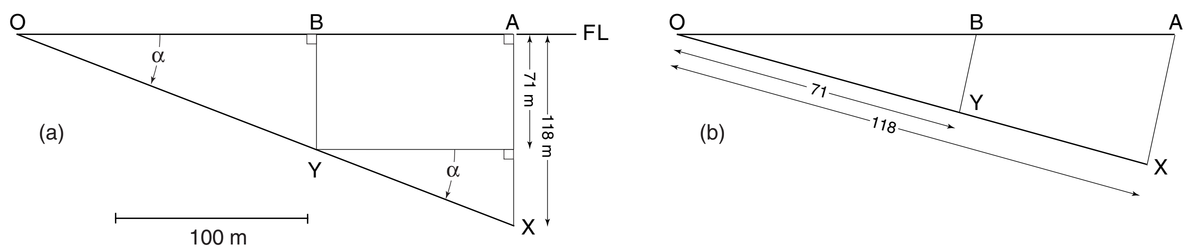

Figure 4.7: Linear interpolation: (a) folding line; (b) scaled line.

In practice two problems arise with this construction. First, the map scale may be such that the depths to X and Y are difficult to plot accurately. Second, if the angle of dip, true or apparent, is small, locating point Y commonly involves a small angle intersection which is subject to a large error. An alternative method minimizes both these difficulties.

Construction II

1. At a convenient but arbitrary angle draw a line from O oblique to the map line OA (Fig. 4.7b). The exact angle does not matter, but it should generally be modest (neither very small or very large).
2. Locate two point on this line:
   (a) Point X at a distance of $\Delta h_A = 296 - 225 = 118$ units.
   (b) Point Y at a distance of $\Delta h_B = 296 - 225 = 71$ units.
3. Choose an arbitrary scale so that distance OX is roughly equal to OA. With a millimeter or engineers trianglar scale this is easily accomplished. Using this scale plot point X at a distance of 118 units and point Y at a distance of 71 units.
4. Connect points A and X and then draw a parallel line through Y to locate point B on OA. With an appropriately chosen scale the angles at A and X will approximately equal and they will be large if α is small.

In this construction triagles OAX and OBY are similar. Therefore as Y divides OX in the ratio $\Delta h_B/\Delta h_A = 71/118$, and so too B divides OA in this same ratio.

The location of the intermediate point B on line OA can also be found by calculating the distance D_{OB} knowing the distance D_{OA}. In Fig. 4.7a OBY and OAX are right triangles and therefore

$$\frac{\Delta h_B}{\Delta h_A} = \frac{D_{OB}}{D_{OA}} \quad \text{or} \quad D_{OB} = D_{OA}\left[\frac{\Delta h_B}{\Delta h_A}\right]. \tag{4.2}$$

In the example problem, distance D_{OY} divides D_{OX} in the ratio $\Delta h_B/\Delta h_A = 71/118$, then B also divides the OA in this same ratio. That is

$$D_{OB} = 300(71/118) = 181 \text{ m},$$

and we can then locate point B on map line OA using the map scale.

There are two situations where such an analytical solution is useful. First, if the locations of points O and A have been determined using modern electronic surveying techniques and are therefore are very accurately known *and* a similar accuracy for the location of point B is required, then the graphic methods are probably inappropriate.

Second, if several related interpolation problems have to be solved routinely then even if great accuracy is not needed an answer can be obtained quickly with a calculator.

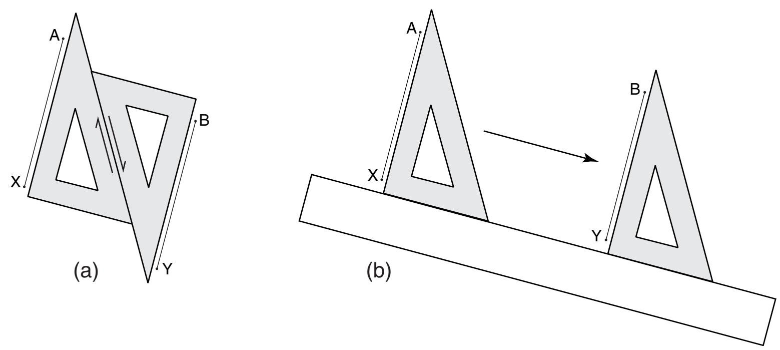

Figure 4.8: Simple methods of drawing parallel lines.

4.5 PARALLEL LINES

In several constructions we need to accurately draw parallel lines. Using a protractor to measure the orientation of the first line AX and then plotting the second line BY using this measured angle is not satisfactory because small errors are inevitable and the lines will not be exactly parallel. There are several alternative ways of drawing such lines more accurately.

1. The easiest way is to use a drafting machine.
2. A T-square and an adjustable triangle on a drafting board is almost as effective.
3. A specialized drafting tool called a *parallel glider* (essentially a straight edge attached to a pair of small wheels) can also be used. This device has the advantage of portability.
4. There are simple, serviceable alternatives.
   (a) Using two identical triangles, place the side of one triangle along the line AX. With the second triangle in contact along their hypontenuses slide this trangle and draw the parallel line BY (Fig. 4.8a).
   (b) Using a triangle and a straight edge, place one side of the triangle along the line AX. Then place a straight edge along the base of the triangle, and shift it along this base and draw the required parallel line BY (Fig. 4.8b).

4.6 THREE-POINT PROBLEM

With the accurate location of such an intermediate point with known elevation established by linear interpolation on a line with known end points we are now prepared to determine the strike and dip of a plane from three general points whose map locations and elevations are known.

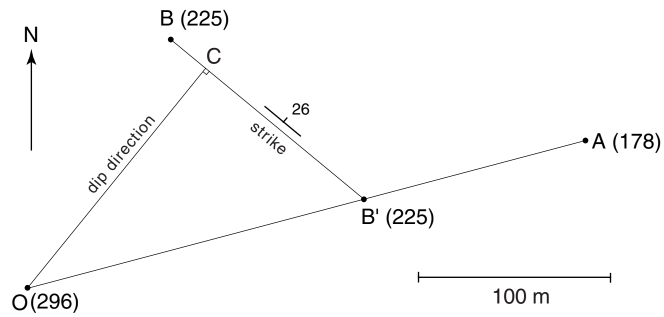

Figure 4.9: Strike and dip from three points on a plane.

Problem

- From the map location of points O, A and B on a plane and their elevations $h_O = 296$ m, $h_A = 178$ m and $h_B = 225$ m, determine the attitude of the plane (Fig. 4.9).

Construction

1. Label the highest point O which serves as a local origin, the lowest point A and the intermediate point B. Draw line OA.
2. Locate point B' with elevation $h_B = 296$ m between points O and A by linear interpolation (as in Fig. 4.7). Line BB' is then a line of strike.
3. From O a perpendicular line intersects BB' at point C. Line OC is then the dip direction. As in Fig. 4.5 a vertical section parallel to this direction can be constructed giving the dip of the plane, or Eq. 4.1 can be used to calculate it.

Answer

- The attitude of the plane is N 50 W, 26 N.

The three-point problem may also be viewed as an exercise in finding the true dip and strike from two apparent dips. First, the inclinations in directions OA and OB are found from the map distances D_{OA} and D_{OB} and elevation differences Δh_A and Δh_B either by the graphical construction of Figs. 1.11 or 1.12 or with

$$\alpha_A = \arctan(\Delta h_A / D_{OA}) \quad \text{and} \quad \alpha_B = \arctan(\Delta h_B / D_{OB}). \tag{4.3}$$

In the example problem

$$\alpha_A = \arctan(118/300) = 21.5^\circ \quad \text{and} \quad \alpha_B = \arctan(71/150) = 25.3^\circ.$$

If, as in this case, the distances are measured to the nearest meter, then the tenths of a degree are significant. The dip and strike can then be found with the constructions of Figs. 1.12, 1.13 or 1.15, or it may be calculated using Eqs. 1.10 and 1.11.

A closely related problem is to determine the elevation of a fourth point on the plane with known dip and strike from its map location.

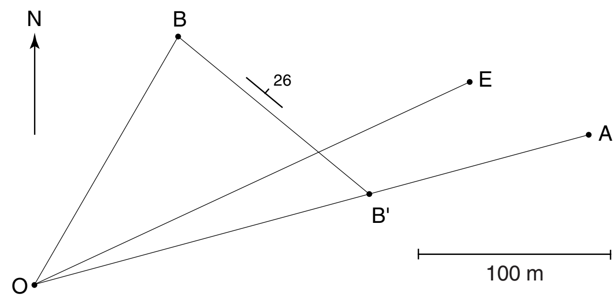

Figure 4.10: Elevation of a fourth point on the plane.

Problem

- What is the elevation of point E on the plane (Fig. 4.10)?

Construction

1. From the known dip of the plane, determined the apparent dip in the direction OE graphically or with Eq. 1.5 or 1.8. The result is $\alpha_D = 23.5°$.

2. With the apparent dip α_E in the direction OE, the elevation difference between O and E can be found graphically or computed from

$$\Delta h_E = D_{OE} \tan \alpha_E. \tag{4.4}$$

   With $D_{OE} = 250$ m, then $\Delta h_E = 109$ m.

3. The elevation of the fourth point E is

$$h_E = h_O - \Delta h_E = 296 - 109 = 187 \text{ m}. \tag{4.5}$$

The three-point and related problems can also be solved analytically. As in the case of interpolation these may be used when greater accuracy is required or a number of problems have to be solved quickly. These are treated in some detail in §7.8.

4.7 STRUCTURE CONTOURS

It is often useful to depict an inclined plane on a map with *structure contours*, that is, by a series of parallel lines of equal elevation drawn on its surface at a fixed interval, each line representing a specified vertical distance from an established datum. To draw these contours a series of points must be interpolated corresponding to multiples of the contour interval. Again there are two ways of constructing such structure contours, corresponding to the two methods of graphic interpolation.

Problem

- Represent the inclined plane of Fig. 4.9 by structure contours with a contour interval of 20 m.

Construction I

1. As in Fig.l 4.7a, construct a section with the horizontal line OA as sl FL and along the vertical line AX plot a series of points with elevations 280–160 m at intervals of 20 m using the map scale. Note that horizontal line OA has an elevation of 296 m so that the 280 m contour is just 16 m below it (Fig. 4.11a).

2. From these elevation points draw a series of horizontal lines to intersect the inclined line OX.

3. Project these points back to OA to intersect it at 90°.

4. From these points add the structure contours parallel to the known strike direction.

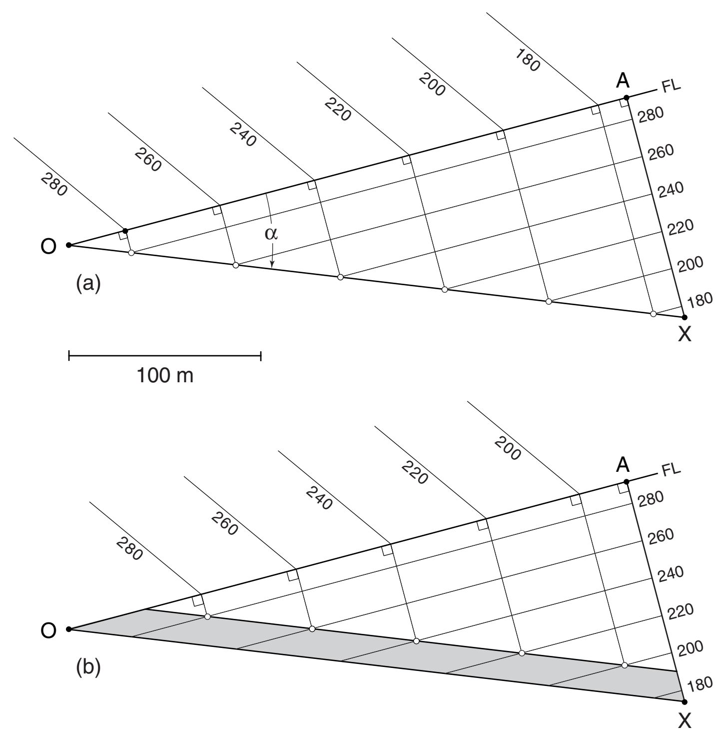

Figure 4.11: Structure contours on a dipping stratum: (a) contours on the bottom of the formation; (b) contours on top of the formation.

In some cases structure contours may be required on both top and bottom of an inclined layer. Given that the contours of Fig. 4.11a represent the bottom of such a layer, we now require contours on its upper bounding plane (Fig. 4.11b). The construction proceeds exactly as before, except now the inclined line representing the top is used instead.

This method suffers from the same problems noted in Fig. 4,7a, and the use of the scaled line avoids small-angle. It also requires fewer steps. Further, contours above or below the two known points can also be easily established by *linear extrapolation*.

Construction II

1. Draw a line at a convenient angle through point O, and locate point X on this line at a distance $D_{OX} = h_O - h_X = 118$ units using a convenient scale (Fig. 4.12a).

2. As before locate a series of points along this line corresponding to the actual contour values. It is possible to start measuring from the point O, but a useful strategy is to shift the scale so point O matches its elevation on the scale. In this example $h_O = 296$, so the scale is shifted 4 units to the left. Then all points at 20 unit increments can be easily marked. It is also easy to include contours beyond the end points O and A.

3. Project these point back to QA parallel to the line AX.

4. Then as before, draw the contours parallel to the known strike.

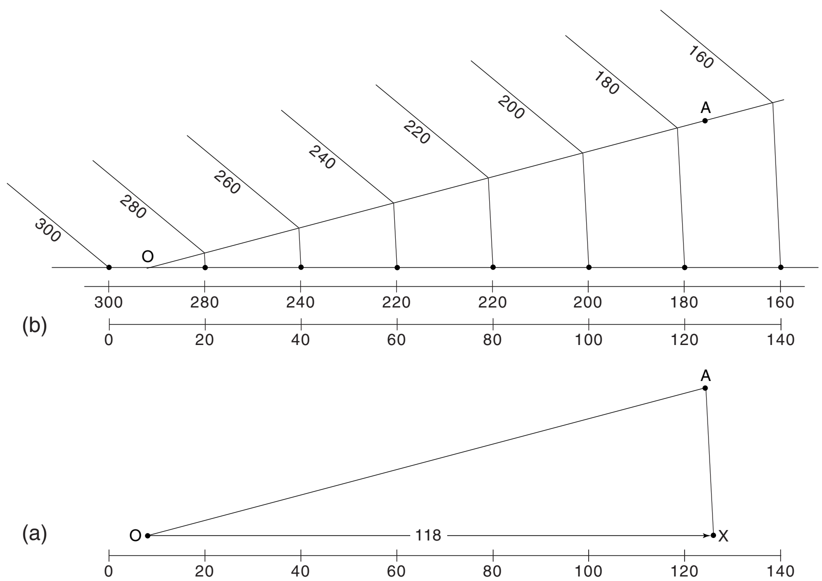

Figure 4.12: Structure contours: (a) the scaled line; (b) structure contours.

Instead of locating these contours graphically, it may be easier and faster, especially for small dip angles, to calculate the map spacing ΔD_{CI} between the contours along a line of true or apparent dip using

$$d_I = \Delta h_I / \tan\delta \quad \textit{or} \quad d_I = \Delta h_I / \tan\alpha, \tag{4.6}$$

where Δh_{CI} is the contour interval. One way of using this result is to set a pair of dividers to this calculated distance and to step off a series of points along a line in the true or apparent dip direction. However, a small error in the original setting will be compounded as the number of contours increases. For example, if the setting has a 1% error, the tenth contour will be 10% in error. A better way is to determine multiples of D_{CI} and plot these distances without moving the scale. There will inevitably be small plotting errors associated with each of these points but these errors will be independent.

4.8 PREDICTING OUTCROP PATTERNS

We may also reverse the processes of determining the attitude of a plane from known points and construct the outcrop trace of an inclined plane from its attitude at a single point. The earth's surface is represented on a map by topographic contours. As we have seen, structural surfaces can be similarly represented by structural contours. If both are represented by contours with the same interval and datum, points of intersection of corresponding contour lines represent points common to the topographic and structure contours, that is, outcrop points of the structural plane.

The technique for accomplishing this is illustrated with the block diagram (Fig. 4.12). Knowing the attitude of the structural plane at a single point O, a vertical section perpendicular to the strike direction is established and topographic contours are added to this section. Starting at the known point on this section, the trace of the dipping plane is then drawn. The intersection of this inclined line and the topographic contours fixes the locations of each structural contour (Points 1, 2, 3, 4).

Projecting these contours in the strike direction then locates points of intersection with the topographic contours on the earth's surface. The outcrop trace is completed by connecting these points. Note that not all of the structure contours are used; the contour associated with Point 4 remains totally underground. In other cases, the contours may be completely in the air.

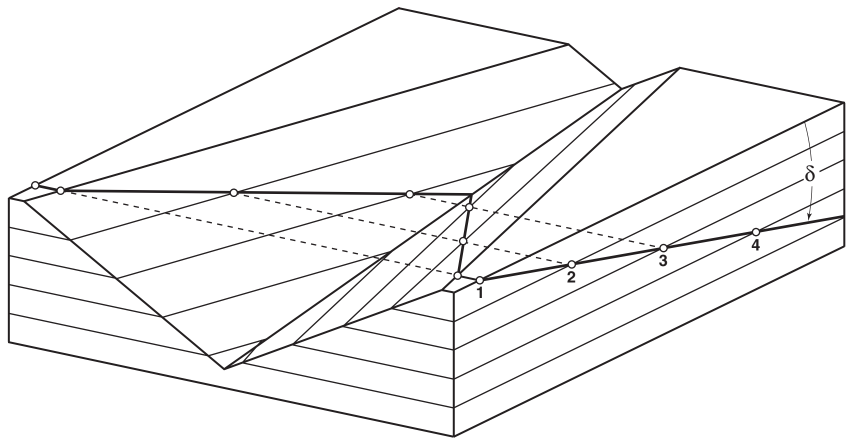

Figure 4.13: Geometric basis for predicting the outcrop trace of an inclined plane.

Problem

- Given a topographic map and a single outcrop point Z on a structural plane whose dip is 20° due north, construct the outcrop trace of the plane in the map area.

Construction

1. As in Fig. 4.10 construct a series of structure contours representing the inclined plane on the section showing the true dip angle. The contour interval must be the same as topographic map, that is 10 m, and the 1260 m structure contour must pass through known point Z (Fig. 4.13).
2. Each intersection of a structure contour with its matching topographic contour is an outcrop point, and these should be marked distinctly. An easy way to avoid mismatching these contours is to start at the known point, drop down one structure contour and one topographic contour and mark the point. Repeat this until you run out of topographic contours and then reverse the direction and move up one contour at a time. Continue this up and down process until all points have been marked.
3. Complete the outcrop trace of the plane by joining successive outcrop points. This line must cross at and only at these marked points. If the contour spacing is wide, the outcrop trace can usually be sketched across the gap by visual interpolation.

Drawing the outcrop trace should be something more that an exercise in connecting points, as in a child's work book; they should certainly not be straight lines. Especially at breaks in slopes, in valley bottoms and on ridge crests it may be necessary to interpolate intermediate structure and the topographic contours, at least mentally, in order to achieve the desired sensitivity to the effects of topography on the outcrop pattern.

If both the upper and lower bounding planes of a layer are to be shown it is a simple matter to add the second boundary to the section using the thickness of the layer, then constructing a second set of structure contours and repeat the procedure for the outcrop trace of this other boundary.

Z Z

Figure 4.14: Outcrop pattern of an inclined plane exposed at point Z (SE corner of map).

4.9 EXERCISES

1. Determine the attitude of the mapped unit of Fig. 4.14a. With this result view the map in a down-dip direction and, in combination with a visualization of the topography, try to see the unit as a layer in edge view. The topography of Fig. 4.14b is identical. With your visualization of the first map as a reference, now try to look down the dip of this layer, and estimate its different attitude and thickness. Check your results.

2. With the following information and the topographic map of Fig. 4.15 (see the end of the book for a full-scale version), construct a geologic map. The base of a 100 m thick sandstone unit of lower Triassic age is exposed at point A; its attitude is N 70 W, 25 S. Point B is on the east boundary of a 50 m thick, vertical diabase dike of Jurassic age; its trend is N 20 E. At point C, the base of a horizontal Cretaceous sequence is exposed and at point D the base of a conformable sequence of Tertiary rocks is present.

Figure 4.15: Determine thickness from map.

Figure 4.16: Geologic map.