Chapter 3

LINES & INTERSECTING PLANES

3.1 DEFINITIONS

Line: the geometric element generated by a moving point; it has only extension along the path of the point. Lines may be rectilinear (straight) or curvilinear (curved). Only straight lines are treated here.

Plunge: the vertical angle measured downward from the horizontal to a line (Fig. 3.1a).

Pitch: the angle between the strike direction and a line in a specified plane (Fig. 3.1b). Rake is synonymous.

Trend: the horizontal direction of the vertical plane containing the line, specified by its bearing or azimuth.

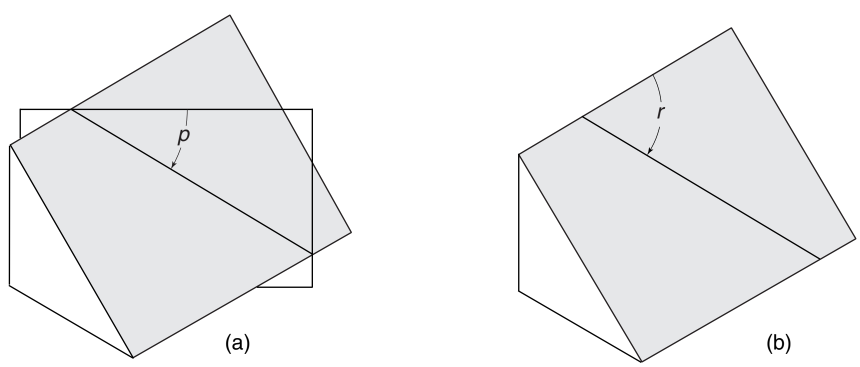

Figure 3.1: Inclination of a line: (a) plunge p; (b) pitch r.

3.2 LINEAR STRUCTURES

There are two types of structural lines. They may exist in their own right, such as the long axes of mineral grains or streaks of mineral aggregates; elongate rock bodies and drill holes may also be considered linear for some purposes. Other lines occur in conjunction with structural planes; examples include striations on fault surfaces, mineral lineation on foliation planes and lines formed by the intersection of planes.

The orientation of a line in space is specified by its trend and plunge. As with planes, there is a set of map symbols for structural lines, also with three parts.

1. A *trend line*.

2. An *arrowhead* giving the direction of downward inclination.

3. A *plunge angle* written near the arrowhead.

The arrows should be uniform in length and long enough so that its trend can be accurately measured on the map. Because its length is not scaled, this symbol is not a vector.

The most common symbols are shown in Fig. 3.2. Special symbols may be invented when needed, and all symbols used must be explained in the map legend.

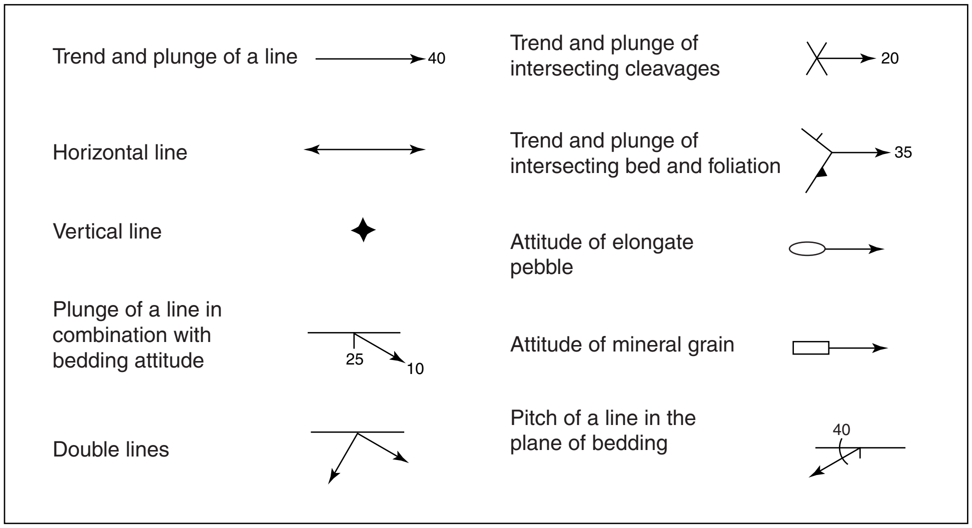

Figure 3.2: Map symbols for structural lines.

The plunge and trend of a line may also be written out. The notation has two forms depending on whether the trend is expressed as a bearing or an azimuth.

1. The plunge angle is followed by the trend expressed as a bearing, as in 30, S 45 W, meaning that the line plunges 30° toward S 45 W.

2. The trend is given as an azimuth, as in 30/225. The order is sometimes reversed as 225/30 and expressing the azimuth with three digits even if this requires leading zeros avoids any possible confusion.

Again, the difference depends of the type of compass used and on personal preference. The azimuth form is particularly useful for computer processing of orientational data.

3.3 PLUNGE OF A LINE

A plunging line is accurately depicted in the vertical section parallel to the trend of the line. If a line is to be depicted on any other section, the angle of *apparent plunge* must be used. The need for such an angle arises if an inclined drill hole and the rock units it penetrates are to be shown on a section oblique to the line of section (Fig. 3.3). Because such displays inevitably involve distortions, it is better to project over small distances and small angles if possible.

This angle is analogous to apparent dip except that the apparent plunge p' is always greater than the true plunge p. When the section line is parallel to the trend of the line, the true plunge is shown and this is the minimum value. If the section is perpendicular to the trend of the line, then the apparent plunge is 90° and this is its maximum value.

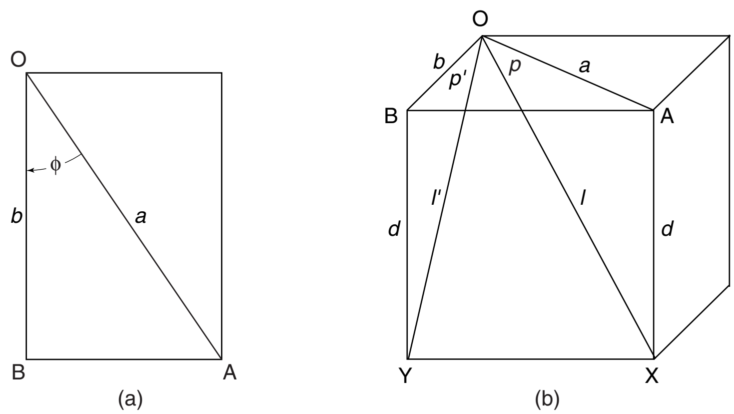

Figure 3.3: Plunge p and apparent plunge p'.

Problem

- Project the inclined drill hole whose attitude is 30/150 onto a north-south section (Fig. 3.4a).

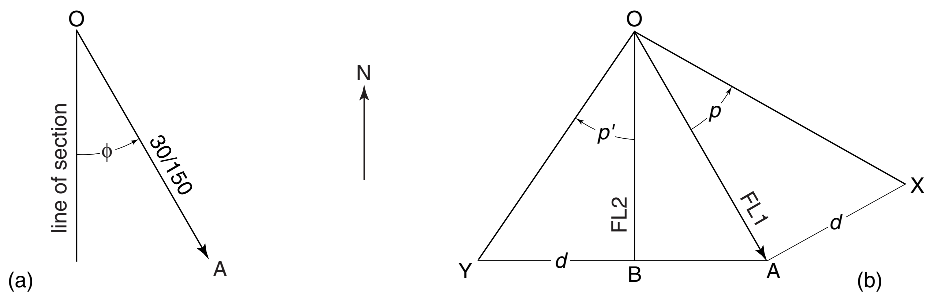

Figure 3.4: Apparent plunge p': (a) map; (b) construction.

Construction

1. In map view, draw a line of section parallel to the trend of the hole (Fig. 3.4b). With this line as *FL1* draw a section showing the true plunge.

2. With the angle of true plunge p find the depth d to point X at some convenient horizontal distance OA, or use the actual depth at the end of the hole to fix OA.

3. Project this surface point A back to the section line to locate point B.

4. With the line of the section as *FL2*, locate point Y at the same depth d below point B. The $\angle BOY$ is the apparent plunge p'.

Answer

- The inclination of the drill hole the vertical north-south section is $p' = 35°$.

The trigonometric expressions for solving this same problem can be derived from the map of Fig. 3.3a

$$b/a = \cos\phi$$

and the block diagram of Fig. 3.3b

$$a = d/\tan p, \quad b = d/\tan p'.$$

Substituting the expressions for a and b in the first equation and rearranging then gives

$$\boxed{\tan p' = \tan p/\cos\phi} \tag{3.1}$$

From the previous problem $p = 30°$ and $\phi = 35°$, and therefore $p' = 35°$.

When projected to a vertical plane of section, any original length l measured along the plunging line is shortened to l'. Again from Fig. 3.3b

$$d = l/\sin p \quad \text{and} \quad d = l'/\sin p'.$$

Equating these two and rearranging yields

$$\boxed{l' = l(\sin p\,/\sin p')} \tag{3.2}$$

Our only concern in the remainder of this chapter is with lines lying in planes. For such lines the plunge angle is, in effect, an apparent dip. Therefore, all the graphical techniques of §1.5 are applicable with little modification to problems involving the relation of the plunge and trend of a line and the dip and strike of the plane. For the same reason, an expression for the plunge can be obtained directly from Eq. 1.4, which we rewrite as

$$\boxed{\tan p = \tan\delta \sin\beta} \tag{3.3}$$

where p is the plunge and, as before, δ is the dip of the plane and β is the structural bearing measured from the strike direction.

3.4 PITCH OF A LINE

The pitch is measured in a plane containing the line (Fig. 3.1b). Therefore it may range in value from $r = 0°$ when the line is horizontal, to $r = 90°$ when the line is in the dip direction. In describing pitch it is necessary only to give the angle and the direction in which the acute angle faces. For example, 35 N means that the pitch angle $r = 35°$ is measured downward from the northern end of the strike line.

In the field, the pitch of a line on an exposed plane is determined by first marking a horizontal line on the inclined plane and then measuring the angle between this line of strike and the linear structure with a protractor. If, however, the plane is not well exposed, the pitch angle may be determined from the dip of the plane and the structural bearing of the line. First, we treat the problem of determining the pitch from the known dip of the plane and a specified structural bearing. We do this by converting β to r.

Problem

- What is the pitch angle of a line with structural bearing $\beta = 35°$ on a plane with dip $\delta = 40°$?

Approach

- In the map view of an inclined plane containing a line we see directly the angle of the structural bearing β, which is the orthographic projection of the pitch angle r to the horizontal plane of the map (Fig. 3.5a). In order to determine this angle, we must obtain a direct view of the inclined plane and the line it contains. This is done by rotating the structural plane into a horizontal position using the strike direction as a folding line (Fig. 3.5b). This procedure can be usefully illustrated by unfolding a small paper model.

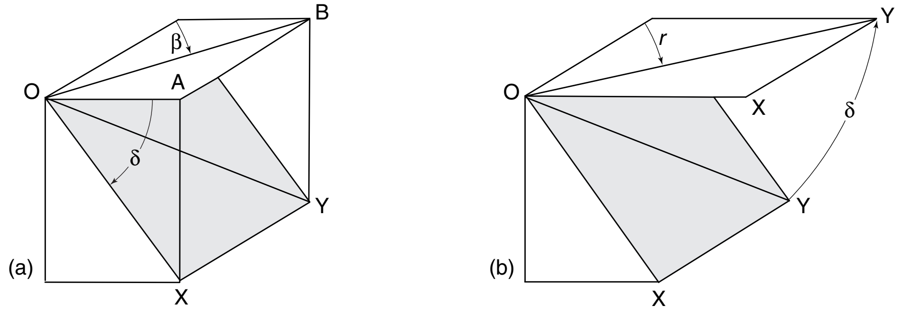

Figure 3.5: Pitch: (a) projection of plane; (b) direct view of plane.

Construction

1. Draw a map showing a strike line, the trend of the inclined line and a line in the true dip direction. At a convenient distance OA draw a second line of strike AB (Fig. 3.6a).
2. With the dip line OA as *FL1* construct a vertical section showing the angle of true dip and determine the depth d of point X directly beneath the surface point A.
3. With the strike line through O as *FL2* using a compass rotate the inclined plane through the angle of dip thus locating points X and Y on the now horizontal structural plane.
4. In this upturned plane the angle line OY makes with the strike direction is the pitch angle r.

Answer

- The pitch of the line in the plane is $r = 42°$.

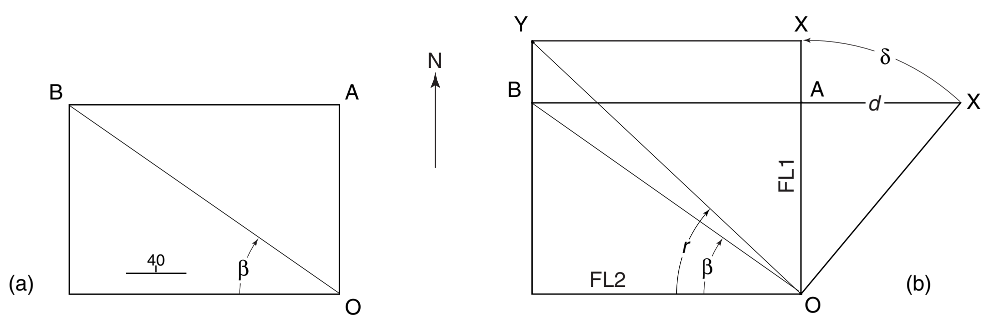

Figure 3.6: Finding pitch: (a) map; (b) construction.

A more useful version of this problem is to determine the plunge of the line whose pitch has been measured and this requires the conversion of r to β. The construction is the reverse of that used in Fig. 3.6.

Problem

- What is the plunge of a line whose pitch $r = 42°$ on a plane with dip $\delta = 40°$?

Construction

1. Draw a direct view of the plane representing the pitch by the line OY which makes an angle of $r = 42°$ with the strike direction through O (Fig. 3.7a).

2. With the dip line as *FL1* construct a vertical section showing the angle of true dip.

3. In this section and using a compass, rotate point X downward to fix its location at depth d below point A. Draw the line AB as a second line of strike.

4. Construct two lines perpendicular to the dip line, the first through A to intersect the trend line at A'.

5. The angle between the strike line through O and line OB is the structural bearing β.

6. With β we can find the plunge using the same construction used to find the apparent dip (Fig. 3.7b).

Answer

- The bearing of the line is $\beta = 35°$ and its plunge is $p = 25°$.

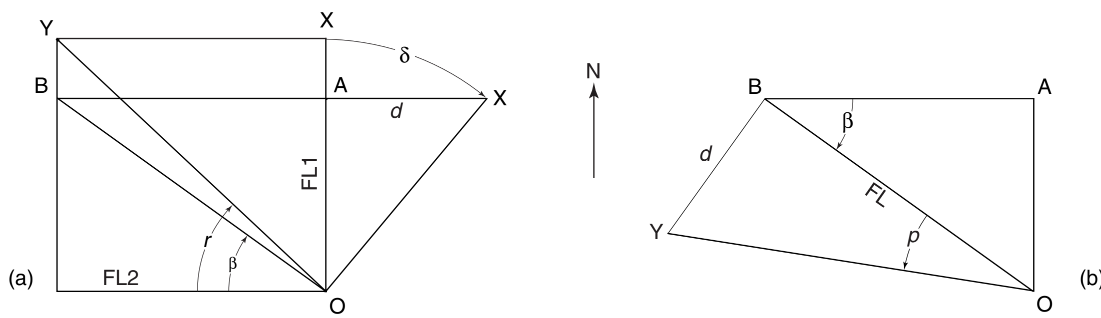

Figure 3.7: Finding plunge from pitch.

There are two useful formulas which can be used to solve problems involving pitch angles. The first relates the pitch of a line to its structural bearing and the dip of the plane. From Fig. 3.8

$$w = c\cos\delta, \quad x = c/\tan r, \quad \tan\beta = w/x.$$

Substituting the first two of these in the third and rearranging gives

$$\boxed{\tan r = \tan\beta\,/\cos\delta} \tag{3.4}$$

From the previous problem, $r = 42°$ and $\delta = 40°$. Therefore $\beta = 35°$. The second formula relates pitch to the structural bearing and plunge angle of the line. Again from Fig. 3.8

$$x = l'\cos\beta, \quad l = l'/\cos p, \quad \cos r = x/l.$$

Substituting the first two of these into the third gives

$$\boxed{\cos r = \cos p\cos\beta} \tag{3.5}$$

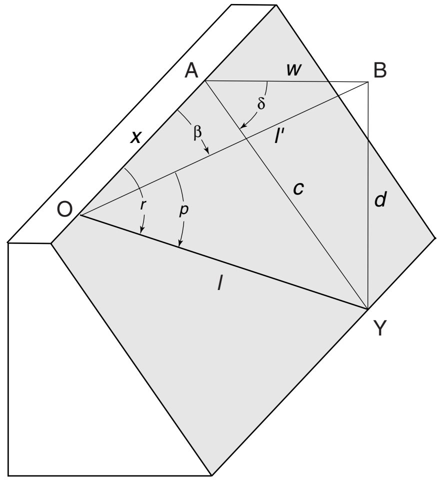

Figure 3.8: Parameters used in expressions for the angle of pitch.

3.5 INTERSECTING PLANES

Two non-parallel planes intersect in a line and in a number of situations it is important to determine the attitude of this line.

Before describing the construction, it is useful to visualize the geometry of intersecting planes and it helps to practice with the aid of flattened hands held parallel to the two planes. Folded paper models also can be used with advantage.

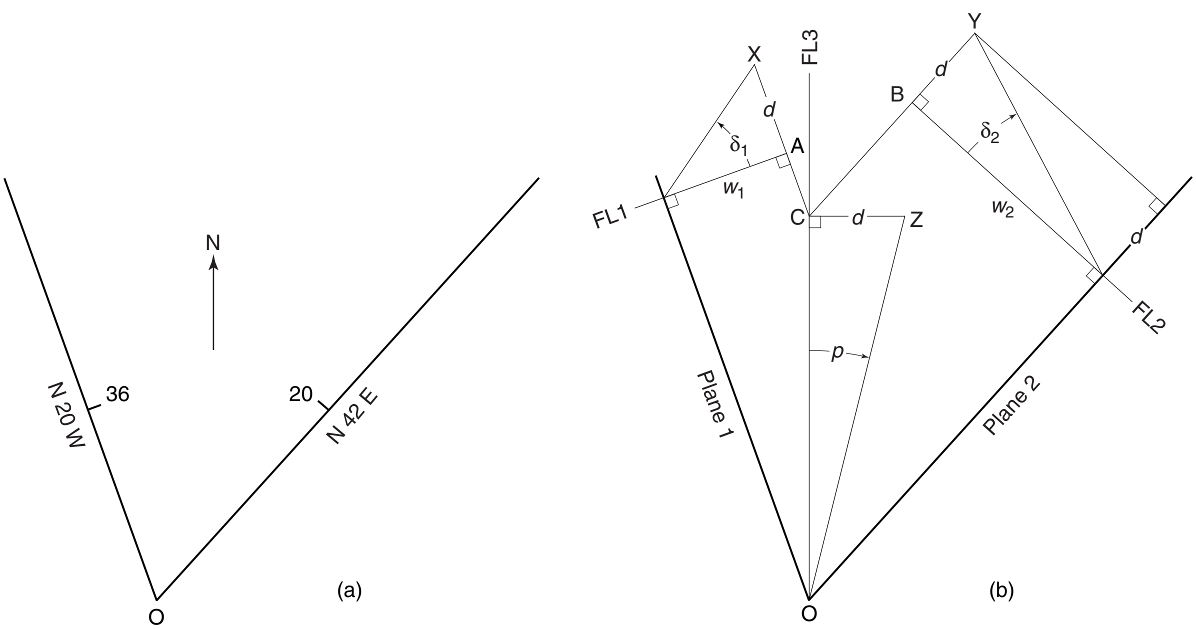

Figure 3.9: Line of intersection: (a) map; (b) construction.

Problem

- What is the attitude of the intersection of Plane 1 (N 20 W, 36 E) with Plane 2 (N 42 E, 20 W)?

Approach

- A view of the map as if one were looking down a V-shaped trough indicates that the line of intersection trends in a northerly direction (Fig. 3.9a). From this view and the previously established relationship between true and apparent dip, we then see that the line of intersection, being common to both planes, cannot have a plunge angle greater than the smaller of the two dip angles.

  The trend of the line of intersection exactly bisects the angle between the strike directions only if the two dip angles are identical. Otherwise the trend of the line will be closer to the strike of the steeper plane. In this problem the plunge of the line of intersection must be less than the 20° dip of Plane 2 and its trend is closer to the strike of steeper Plane 1.

Construction

1. In map view plot a strike line for each of the two planes and label their intersection O which represents one point common to both planes and therefore lies on the line of intersection (Fig. 3.9b).

2. To obtain another point on the line of intersection a second pair of intersecting strike lines at a known vertical distance below the intersecting first pair are needed.

   (a) With *FL1* perpendicular to the strike of Plane 1, construct a vertical section showing the true dip δ_1. Locate surface point A with a convenient distance w_1 from the strike line. Then plot the trace of the inclined plane and determine the depth d to point X below surface point A.

   (b) With *FL2* perpendicular to the strike of Plane 2, construct a second vertical section showing the true dip δ_2. Using the same depth d to point Y, determine the length w_2 and locate the surface point B.

3. Through both points A and B draw a second pair of lines parallel to the strike directions of the respective planes intersecting at point C. Direction OC represents the trend of the line of intersection which can be measured.

4. With *FL3* perpendicular to OC, construct a vertical section again using the same depth d to locate the point Z below at the surface point C. Vertical $\angle COZ$ is the angle of plunge p.

Answer

- The attitude of the line of intersection is 14/000.

3.6 COTANGENT METHOD

This orthographic construction may be simplified by adapting the semi-graphical cotangent method used in §1.6. As before this is equivalent to choosing the depth $d = 1$.

Problem

- Find the intersection of the planes with dip lines D_1(36/070) and D_2(20/312) (Fig. 3.10a).

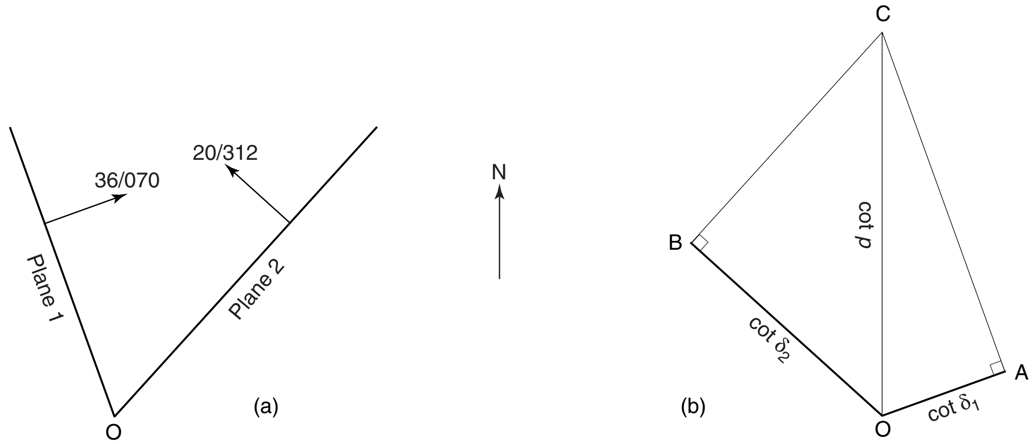

Figure 3.10: Line of intersection by cotangent method.

Construction

1. Plot rays from a common point O in each of the dip directions (Fig. 3.10b).
2. Using a convenient scale measure lengths $OA = \cot\delta_1 = 1/\tan\delta_1 = 1.37638$ and $OB = \cot\delta_2 = 1/\tan\delta_2 = 2.74748$ along the respective rays.
3. Through both A and B draw lines perpendicular to the respective rays. The point C at their intersection is a second point on the line of intersection. Line OC fixes its trend and its length is $\cot p = 4.0$.

Answer

- The line of intersection has a plunge of $p = \arctan(1/4.0) = 14°$ and its trend is 000. This is the same found by the full graphic construction of Fig. 3.9b.

3.7 STRUCTURE CONTOURS

So far we have been concerned only with the *orientation* of the line of intersection. Its *location* is an additional important property.

By definition points on a line of strike have the same elevations and there are an infinite number of such lines on any inclined plane. When the elevation is known or specified for a particular strike line then it becomes a *structure contour* on the plane. The map location of a point on a line of intersection requires two such contour lines, one on each plane.

If the elevations of the two strike lines are identical, then point O of the previous construction represent a point in space on the line. On the other hand, if the two structure contours do not have the same elevations then an auxiliary construction is needed to find a point which is exactly on the line.

Problem

- Given the map location of a point on plane A with attitude N 25 W, 20 W at an elevation of $h_A = 200$ m and the location of point B on a second plane at an elevations of $h_B = 150$ m with attitude N 48 E, 30 S, locate the line of intersections in space (Fig. 3.11).

Approach

- Because they have different elevations, structure contours through A and B cross on the map but do not intersect in space. We need to locate a second contour with the same elevation of one of these and there are two choices: locate the 150 m contour for Plane A, or the 200 m contour for Plane B.

Construction

1. To find an intersection we need a structure contour on one of the planes with the same elevation as the other. Our choice is to construct the structural contour on plane B at the same elevation as the plane at A.

2. With a FL perpendicular to the strike of plane through B draw the inclined trace of the plane on the vertical section. Note that this trace must be projected *upward*, not downward.

3. On this trace locate a point at a scaled distance of 50 m higher than B, project this back to the map, and draw in the required structure contour. The intersection of this 200 m contour with the 200 m through A locates the required point O.

4. Using point O, we can then find the attitude of the line of intersection with the construction of Fig. 3.9 or Fig. 3.10.

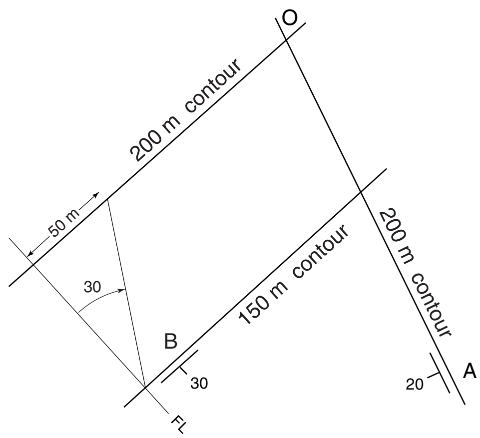

Figure 3.11: Line of intersection when points have different elevations.

3.8 LINE VECTORS

Just as we have represented the orientation of planes with two-dimensional vectors so too can we represent lines by such vectors (see §1.8). The direction of these vectors is the established by trend of the line and its length is equal to $\tan p$. We can then obtain the orientation and length of the intersection vector $\mathbf{I}$ from dip vectors representing the two intersecting planes.

Problem

- From dip vectors $\mathbf{D}_1(36/070)$ and $\mathbf{D}_2(20/312)$ determine the intersection vector $\mathbf{I}$.

Construction

1. Draw the dip vectors from a common point O with lengths $\tan\delta_1 = 0.72654$ and $\tan\delta_2 = 0.36397$ using a convenient scale (Fig. 3.12).
2. Draw a line connecting the end points of these two vectors.
3. The intersection vector (I) is established by drawing the perpendicular from O to this line. This gives its trend; its length is $\tan p$.

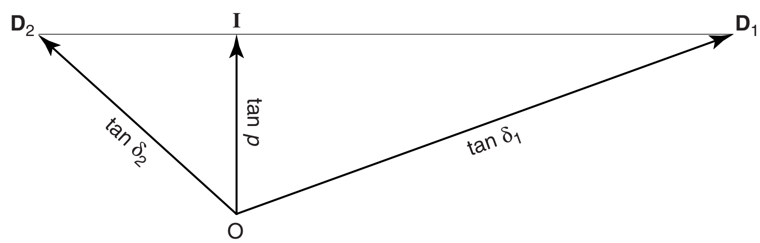

Figure 3.12: Intersection vector from two dip vectors.

Answer

- The measured length of **I** is $\tan p = 0.25$, hence $p = 14°$ and its measured trend is 000.

From this vector method we may also obtain an analytical solution (Fig. 3.13). From the dot product of each of the two dip vectors $\mathbf{D}_1$ and $\mathbf{D}_2$ with the unit vector $\hat{\mathbf{u}}$ in the direction of the unknown intersection vector **I** we have

$$\mathbf{D}_1 \cdot \hat{\mathbf{u}} = \mathbf{D}_2 \cdot \hat{\mathbf{u}}.$$

Using the form of the dot product from Eq. 1.7 gives

$$\tan\delta_1 \cos\phi_1 = \tan\delta_2 \cos\phi_2$$

where ϕ_1 and ϕ_2 are the angles each dip vector makes with $\hat{\mathbf{u}}$. Labeling the angle between $\mathbf{D}_1$ and $\mathbf{D}_2$ ϕ, then $\phi = \phi_1 + \phi_2$, and we have

$$\phi_2 = (\phi - \phi_1).$$

Substituting this and using the identity for the cosine of the difference of two angles

$$\cos(\phi - \phi_1) = \cos\phi\cos\phi_1 + \sin\phi\sin\phi_1$$

we obtain

$$\tan\delta_1 \cos\phi_1 = \tan\delta_2(\cos\phi\cos\phi_1 + \sin\phi\sin\phi_1).$$

Rearranging then yields the expression for the trend of the line measured from $\mathbf{D}_1$

$$\boxed{\tan\phi_1 = \frac{\tan\delta_1}{\tan\delta_2 \sin\phi} - \frac{1}{\tan\phi}} \tag{3.6}$$

The angle of plunge can then be obtained by recasting Eq. 1.11 into a more useful form. Substituting $\beta = 90° - \phi$ and $\alpha = p$ we obtain

$$\boxed{\tan p = \tan\delta_1 \cos\phi_1} \tag{3.7}$$

Using the values from the previous problem $\delta_1 = 36°$ and $\delta_2 = 20°$ and $\phi = 118°$ in Eq. 3.6 gives $\phi_1 = 70.29774°$. Using this Eq. 3.7 gives $p = 13.76330°$. These results are essentially the same as found graphically in Figs. 3.9 and 3.12.

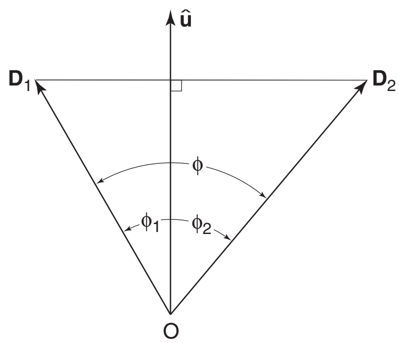

Figure 3.13: Analytical solution of the line of intersection.

3.9 ACCURACY OF TREND DETERMINATIONS

As with dip and strike, the angles of plunge and trend cannot be measured without error, and as before, situations where small measurement errors may become magnified are of special concern.

In measuring the trend of a line on a structural plane it is common practice to align the compass in the direction of the horizontal projection of the line. As a result of an inevitable operator error, the measured trend as given by the angle β' will differ from the true trend given by the angle β. We seek an expression for the *maximum trend error* ϵ_T

$$\epsilon_T = |\beta - \beta'| \tag{3.8}$$

in terms of the angle ϵ_O which the measured line makes with the true line as measured in the inclined plane which results from the maximum operator error. That is, we compare the true trend of a line as given by β with β' found from the pitch angle $(r \pm \epsilon_O)$ on the same plane, where r is the true pitch angle.

There are two cases depending on whether the measured trend is on the down-dip $(\beta' > \beta)$ or on the up-dip $(\beta' < \beta)$ side of the line. The results are show graphically in Fig. 3.14.

1. If $\beta' > \beta$ we find an expression for this error we use Eq. 3.4, giving

   $$\tan\beta = \tan r \cos\delta \quad \text{and} \quad \tan\beta' = \tan(r + \epsilon_O)\cos\delta.$$

   Substituting these into the equation for the tangent of the difference of two angles

   $$\tan(\beta' - \beta) = \frac{\tan\beta' - \tan\beta}{1 + \tan\beta\tan\beta'},$$

   we have (after Woodcock, 1976, p. 352)

   $$\tan\epsilon_T = \frac{[\tan(r + \epsilon_O) - \tan r]\cos\delta}{1 + [\tan(r + \epsilon_O)\tan r]\cos^2\delta}. \tag{3.9}$$

   The graph of this equation for $\epsilon_O = 3°$ shows that a combination of a steep plane and a large pitch angle may result in a large trend error.

2. For the case $\beta' < \beta$ we proceed in a similar way by expressing the maximum trend error associated with the smaller pitch angle $(r - \epsilon_O)$. The resulting formula in this case is

   $$\tan\epsilon_T = \frac{[\tan r - \tan(r - \epsilon_O)]\cos\delta}{1 + [\tan r \tan(r - \epsilon_O)]\cos^2\delta} \tag{3.10}$$

   and the corresponding graph shows that, other things being equal, the maximum trend error in this case is less. This means that repeated measurements will not be symmetrically distributed about the true trend.

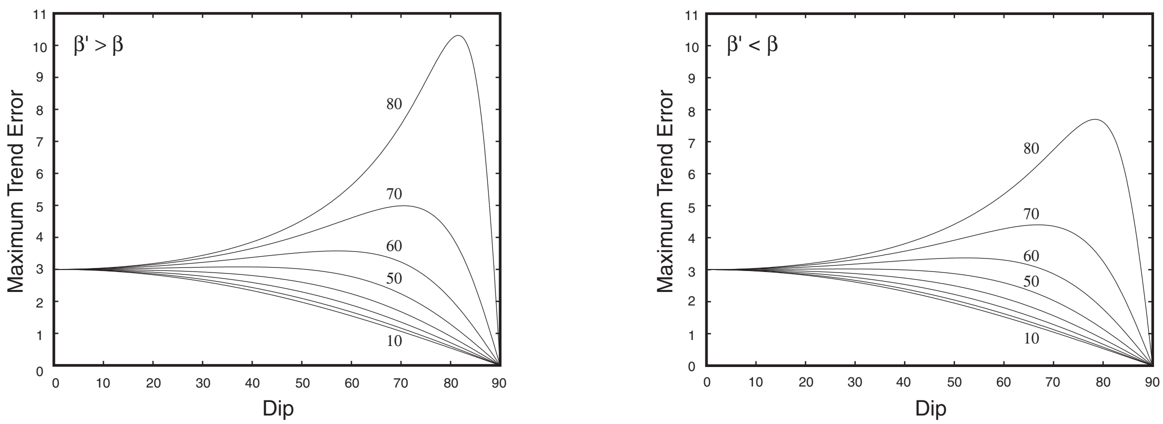

Figure 3.14: Maximum trend error as a function of pitch and dip for $\epsilon_O = 3°$.

To avoid such errors it is advisable to measure the pitch of the line in the dipping plane directly and then determine its attitude either graphically or with the aid of Eq. 3.4.

A second type of error magnification occurs in determining the attitude of the line of intersection of two plane. Because the dip and strike measurements of both planes are subject to their own errors, the derived attitude of the line of intersection will also be in error and this error may be large if the angle between the two planes is small. This problem is treated in detail in Chapter 5.

3.10 EXERCISES

- Using the map of Fig. 3.15, structural plane A is a narrow shear zone whose attitude is N 66 E, 50 S, and plane B is the top of a limestone bed whose attitude is N 22 W, 40 W. Determine the orientation of the line of intersection of these two planes, the pitch of this line in plane B, the surface outcrop point of the line, and the depth at which the line would be found by drilling in the bed of Boulder Creek. (Note that points A and B do not have the same elevation).

Figure 3.15: Vein in Boulder Creek